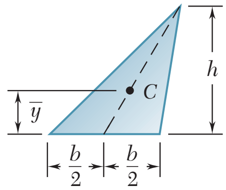
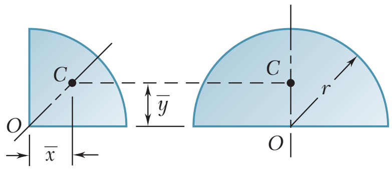
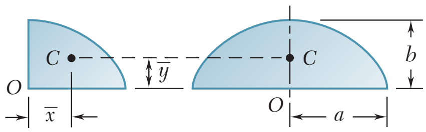
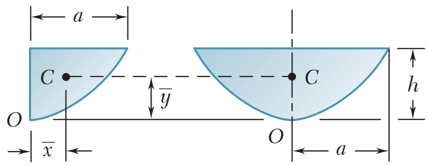
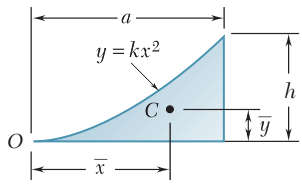
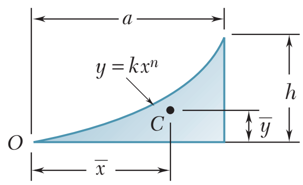
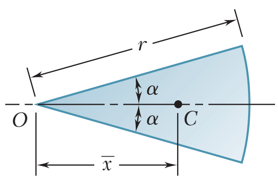
Shape		$\bar{x}$	$\bar{y}$	Area
Triangular area			$\frac{h}{3}$	$\frac{bh}{2}$
Quarter-circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
Semicircular area		0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$
Quarter-elliptical area		$\frac{4a}{3\pi}$	$\frac{4b}{3\pi}$	$\frac{\pi ab}{4}$
Semielliptical area		0	$\frac{4b}{3\pi}$	$\frac{\pi ab}{2}$
Semiparabolic area		$\frac{3a}{8}$	$\frac{3h}{5}$	$\frac{2ah}{3}$
Parabolic area		0	$\frac{3h}{5}$	$\frac{4ah}{3}$
Parabolic spandrel		$\frac{3a}{4}$	$\frac{3h}{10}$	$\frac{ah}{3}$
General spandrel		$\frac{n+1}{n+2}a$	$\frac{n+1}{4n+2}h$	$\frac{ah}{n+1}$
Circular sector		$\frac{2r\sin\alpha}{3\alpha}$	0	αr^2

Fig. 5.8A Centroids of common shapes of areas.

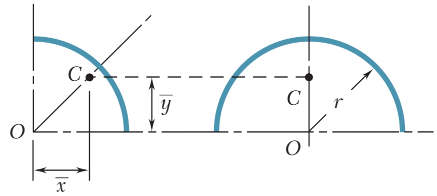
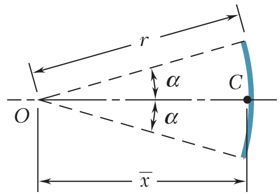
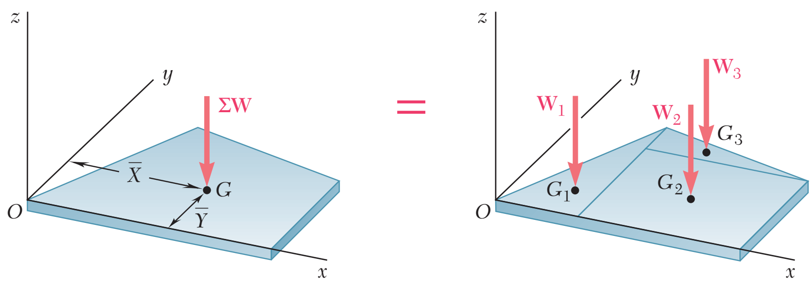
Shape		$\bar{x}$	$\bar{y}$	Length
Quarter-circular arc		$\frac{2r}{\pi}$	$\frac{2r}{\pi}$	$\frac{\pi r}{2}$
Semicircular arc		0	$\frac{2r}{\pi}$	πr
Arc of circle		$\frac{r \sin \alpha}{\alpha}$	0	$2\alpha r$

Fig. 5.8B Centroids of common shapes of lines.

5.5 COMPOSITE PLATES AND WIRES

In many instances, a flat plate can be divided into rectangles, triangles, or the other common shapes shown in Fig. 5.8A. The abscissa $\bar{X}$ of its center of gravity G can be determined from the abscissas $\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_n$ of the centers of gravity of the various parts by expressing that the moment of the weight of the whole plate about the y axis is equal to the sum of the moments of the weights of the various parts about the same axis (Fig. 5.9). The ordinate $\bar{Y}$ of the center of gravity of the plate is found in a similar way by equating moments about the x axis. We write

$$\Sigma M_y: \quad \bar{X}(W_1 + W_2 + \cdots + W_n) = \bar{x}_1W_1 + \bar{x}_2W_2 + \cdots + \bar{x}_nW_n$$
$$\Sigma M_x: \quad \bar{Y}(W_1 + W_2 + \cdots + W_n) = \bar{y}_1W_1 + \bar{y}_2W_2 + \cdots + \bar{y}_nW_n$$



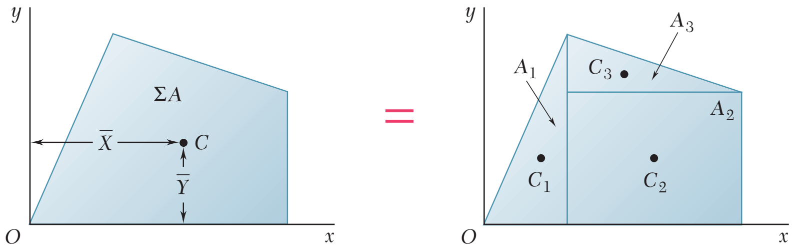
$$\Sigma M_y: \quad \bar{X}\,\Sigma W = \Sigma\,\bar{x}\,W$$
$$\Sigma M_x: \quad \bar{Y}\,\Sigma W = \Sigma\,\bar{y}\,W$$

Fig. 5.9 Center of gravity of a composite plate.

or, for short,

$$\overline{X}\Sigma W = \Sigma\overline{x}W \qquad \overline{Y}\Sigma W = \Sigma\overline{y}W \tag{5.7}$$

These equations can be solved for the coordinates $\overline{X}$ and $\overline{Y}$ of the center of gravity of the plate.



$Q_y = \overline{X}\Sigma A = \Sigma\overline{x}A$
$Q_x = \overline{Y}\Sigma A = \Sigma\overline{y}A$

Fig. 5.10 Centroid of a composite area.

If the plate is homogeneous and of uniform thickness, the center of gravity coincides with the centroid C of its area. The abscissa $\overline{X}$ of the centroid of the area can be determined by noting that the first moment Q_y of the composite area with respect to the y axis can be expressed both as the product of $\overline{X}$ and the total area and as the sum of the first moments of the elementary areas with respect to the y axis (Fig. 5.10). The ordinate $\overline{Y}$ of the centroid is found in a similar way by considering the first moment Q_x of the composite area. We have

$$Q_y = \overline{X}(A_1 + A_2 + \cdots + A_n) = \overline{x}_1A_1 + \overline{x}_2A_2 + \cdots + \overline{x}_nA_n$$
$$Q_x = \overline{Y}(A_1 + A_2 + \cdots + A_n) = \overline{y}_1A_1 + \overline{y}_2A_2 + \cdots + \overline{y}_nA_n$$

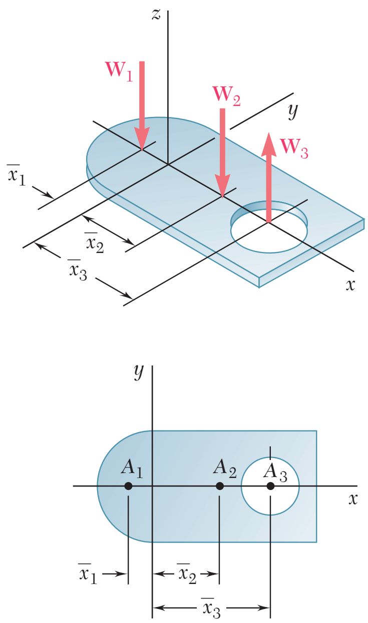
or, for short,

$$Q_y = \overline{X}\Sigma A = \Sigma\overline{x}A \qquad Q_x = \overline{Y}\Sigma A = \Sigma\overline{y}A \tag{5.8}$$

These equations yield the first moments of the composite area, or they can be used to obtain the coordinates $\overline{X}$ and $\overline{Y}$ of its centroid.

Care should be taken to assign the appropriate sign to the moment of each area. First moments of areas, like moments of forces, can be positive or negative. For example, an area whose centroid is located to the left of the y axis will have a negative first moment with respect to that axis. Also, the area of a hole should be assigned a negative sign (Fig. 5.11).

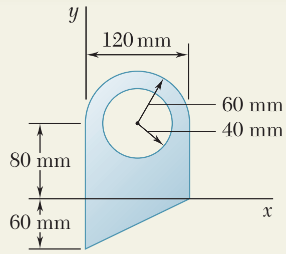
Similarly, it is possible in many cases to determine the center of gravity of a composite wire or the centroid of a composite line by dividing the wire or line into simpler elements (see Sample Prob. 5.2).



	$\overline{x}$	A	$\overline{x}A$
A_1 Semicircle	–	+	–
A_2 Full rectangle	+	+	+
A_3 Circular hole	+	–	–

Fig. 5.11

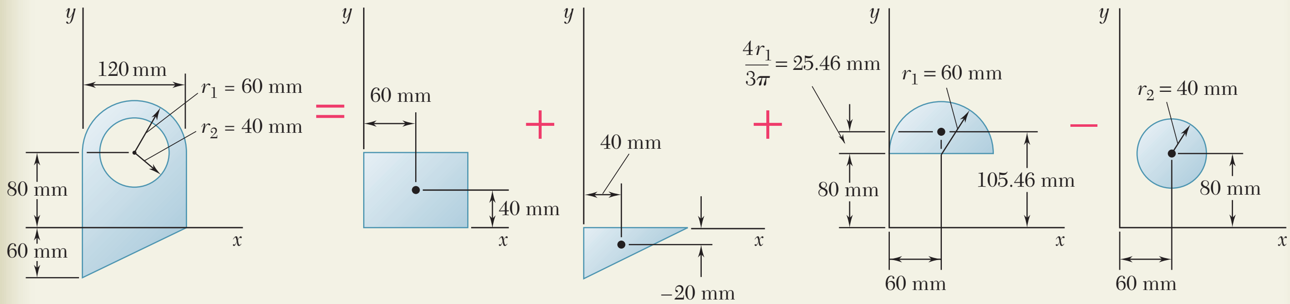
SAMPLE PROBLEM 5.1



For the plane area shown, determine (*a*) the first moments with respect to the x and y axes, (*b*) the location of the centroid.

SOLUTION

Components of Area. The area is obtained by adding a rectangle, a triangle, and a semicircle and by then subtracting a circle. Using the coordinate axes shown, the area and the coordinates of the centroid of each of the component areas are determined and entered in the table below. The area of the circle is indicated as negative, since it is to be subtracted from the other areas. We note that the coordinate $\bar{y}$ of the centroid of the triangle is negative for the axes shown. The first moments of the component areas with respect to the coordinate axes are computed and entered in the table.

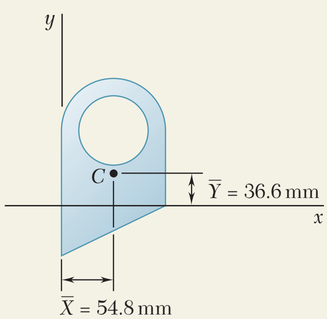


Component	A, mm²	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}A$, mm³	$\bar{y}A$, mm³
Rectangle	$(120)(80) = 9.6 \times 10^3$	60	40	$+576 \times 10^3$	$+384 \times 10^3$
Triangle	$\frac{1}{2}(120)(60) = 3.6 \times 10^3$	40	−20	$+144 \times 10^3$	-72×10^3
Semicircle	$\frac{1}{2}\pi(60)^2 = 5.655 \times 10^3$	60	105.46	$+339.3 \times 10^3$	$+596.4 \times 10^3$
Circle	$-\pi(40)^2 = -5.027 \times 10^3$	60	80	-301.6×10^3	-402.2×10^3
	$\Sigma A = 13.828 \times 10^3$			$\Sigma\bar{x}A = +757.7 \times 10^3$	$\Sigma\bar{y}A = +506.2 \times 10^3$

a. First Moments of the Area. Using Eqs. (5.8), we write

$$Q_x = \Sigma\bar{y}A = 506.2 \times 10^3 \text{ mm}^3 \qquad Q_x = 506 \times 10^3 \text{ mm}^3 \blacktriangleleft$$

$$Q_y = \Sigma\bar{x}A = 757.7 \times 10^3 \text{ mm}^3 \qquad Q_y = 758 \times 10^3 \text{ mm}^3 \blacktriangleleft$$



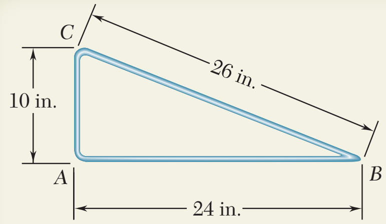
b. Location of Centroid. Substituting the values given in the table into the equations defining the centroid of a composite area, we obtain

$$\bar{X}\Sigma A = \Sigma\bar{x}A: \qquad \bar{X}(13.828 \times 10^3 \text{ mm}^2) = 757.7 \times 10^3 \text{ mm}^3$$

$$\bar{X} = 54.8 \text{ mm} \blacktriangleleft$$

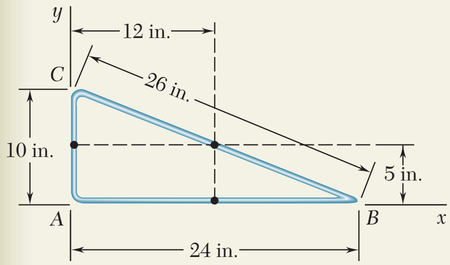
$$\bar{Y}\Sigma A = \Sigma\bar{y}A: \qquad \bar{Y}(13.828 \times 10^3 \text{ mm}^2) = 506.2 \times 10^3 \text{ mm}^3$$

$$\bar{Y} = 36.6 \text{ mm} \blacktriangleleft$$



SAMPLE PROBLEM 5.2

The figure shown is made from a piece of thin, homogeneous wire. Determine the location of its center of gravity.



SOLUTION

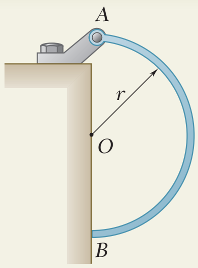
Since the figure is formed of homogeneous wire, its center of gravity coincides with the centroid of the corresponding line. Therefore, that centroid will be determined. Choosing the coordinate axes shown, with origin at *A*, we determine the coordinates of the centroid of each line segment and compute the first moments with respect to the coordinate axes.

Segment	L, in.	$\bar{x}$, in.	$\bar{y}$, in.	$\bar{x}L$, in^2	$\bar{y}L$, in^2
AB	24	12	0	288	0
BC	26	12	5	312	130
CA	10	0	5	0	50
	$\Sigma L = 60$			$\Sigma \bar{x}L = 600$	$\Sigma \bar{y}L = 180$

Substituting the values obtained from the table into the equations defining the centroid of a composite line, we obtain

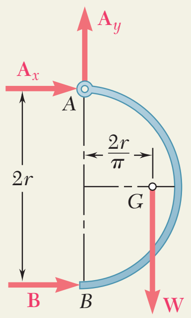
$\bar{X}\Sigma L = \Sigma \bar{x}L$: $\quad \bar{X}(60 \text{ in.}) = 600 \text{ in}^2$ $\quad \bar{X} = 10$ in. ◀

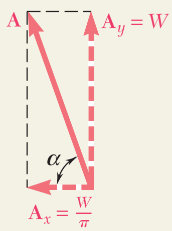
$\bar{Y}\Sigma L = \Sigma \bar{y}L$: $\quad \bar{Y}(60 \text{ in.}) = 180 \text{ in}^2$ $\quad \bar{Y} = 3$ in. ◀



SAMPLE PROBLEM 5.3

A uniform semicircular rod of weight W and radius r is attached to a pin at A and rests against a frictionless surface at B. Determine the reactions at A and B.





SOLUTION

Free-Body Diagram. A free-body diagram of the rod is drawn. The forces acting on the rod are its weight **W**, which is applied at the center of gravity G (whose position is obtained from Fig. 5.8B); a reaction at A, represented by its components $\mathbf{A}_x$ and $\mathbf{A}_y$; and a horizontal reaction at B.

Equilibrium Equations

$$+\circlearrowleft \Sigma M_A = 0: \qquad B(2r) - W\left(\frac{2r}{\pi}\right) = 0$$

$$B = +\frac{W}{\pi} \qquad \qquad \mathbf{B} = \frac{W}{\pi} \rightarrow \blacktriangleleft$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0: \qquad A_x + B = 0$$

$$A_x = -B = -\frac{W}{\pi} \qquad \mathbf{A}_x = \frac{W}{\pi} \leftarrow$$

$$+\uparrow \Sigma F_y = 0: \qquad A_y - W = 0 \qquad \mathbf{A}_y = W \uparrow$$

Adding the two components of the reaction at A:

$$A = \left[W^2 + \left(\frac{W}{\pi}\right)^2\right]^{1/2} \qquad \qquad A = W\left(1 + \frac{1}{\pi^2}\right)^{1/2} \blacktriangleleft$$

$$\tan \alpha = \frac{W}{W/\pi} = \pi \qquad \qquad \alpha = \tan^{-1}\pi \blacktriangleleft$$

The answers can also be expressed as follows:

$$\mathbf{A} = 1.049W \measuredangle 72.3° \qquad \mathbf{B} = 0.318W \rightarrow \blacktriangleleft$$

PROBLEMS

5.1 through 5.9 Locate the centroid of the plane area shown.

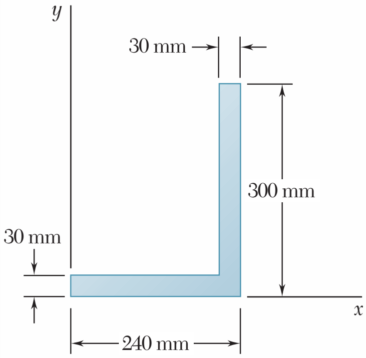


Fig. P5.1

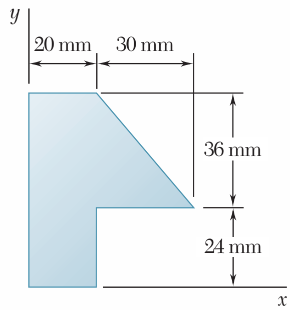


Fig. P5.2

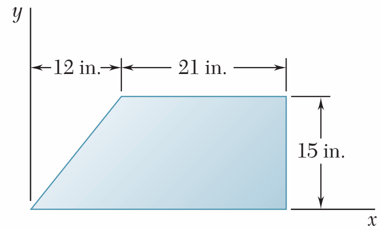


Fig. P5.3

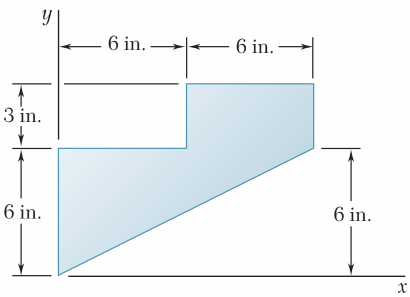


Fig. P5.4

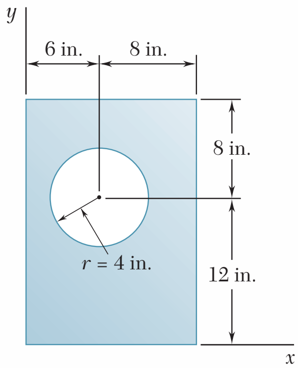


Fig. P5.5

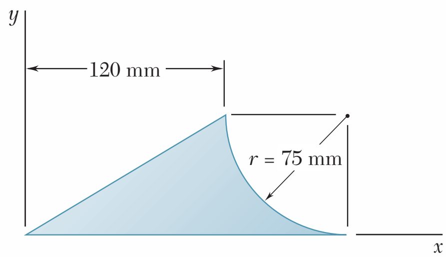


Fig. P5.6

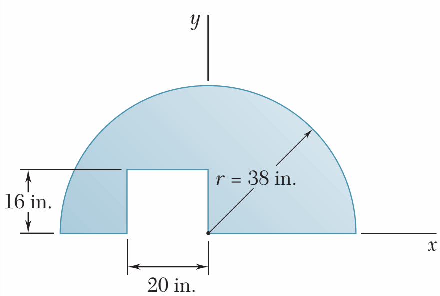


Fig. P5.7

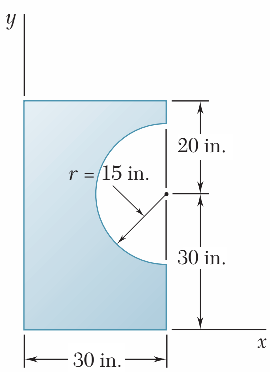


Fig. P5.8

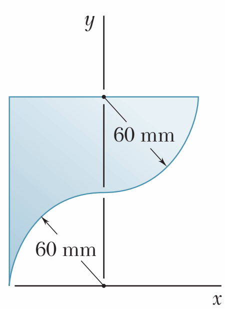


Fig. P5.9

5.10 through 5.15 Locate the centroid of the plane area shown.

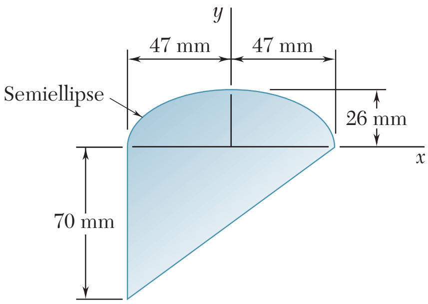


Fig. P5.10

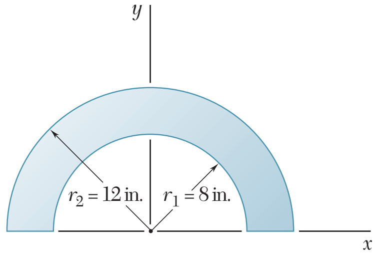


Fig. P5.11

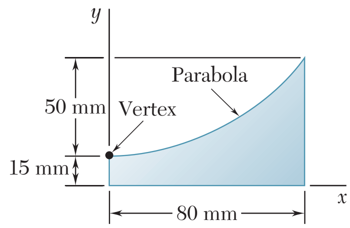


Fig. P5.12

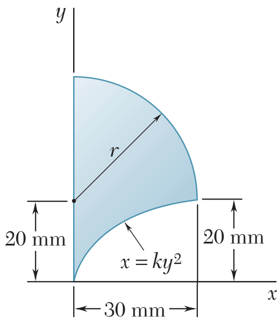


Fig. *P5.13*

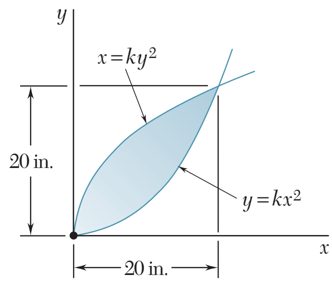


Fig. P5.14

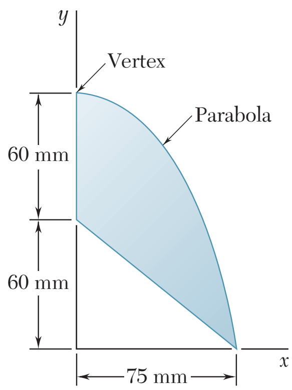


Fig. *P5.15*

5.16 Determine the y coordinate of the centroid of the shaded area in terms of r_1, r_2, and α.

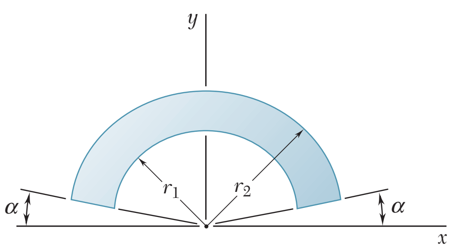


Fig. P5.16 and P5.17

5.17 Show that as r_1 approaches r_2, the location of the centroid approaches that for an arc of circle of radius $(r_1 + r_2)/2$.

5.18 For the area shown, determine the ratio a/b for which $\bar{x} = \bar{y}$.

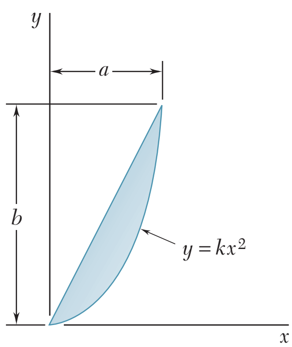


Fig. P5.18

5.19 For the semiannular area of Prob. 5.11, determine the ratio r_2/r_1 so that $\bar{y} = 3r_1/4$.

5.20 A composite beam is constructed by bolting four plates to four $60 \times 60 \times 12$-mm angles as shown. The bolts are equally spaced along the beam, and the beam supports a vertical load. As proved in mechanics of materials, the shearing forces exerted on the bolts at A and B are proportional to the first moments with respect to the centroidal x axis of the red shaded areas shown, respectively, in parts a and b of the figure. Knowing that the force exerted on the bolt at A is 280 N, determine the force exerted on the bolt at B.

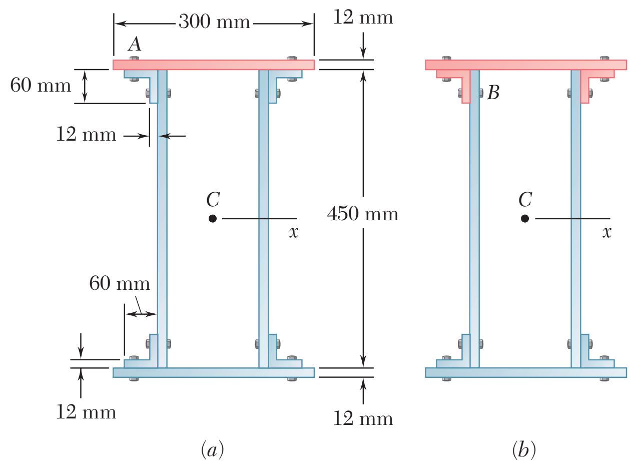


Fig. P5.20

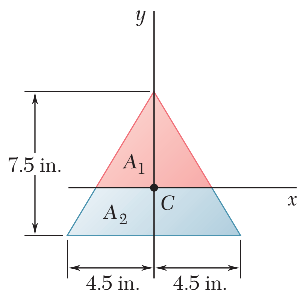


Fig. P5.21

5.21 and 5.22 The horizontal x axis is drawn through the centroid C of the area shown, and it divides the area into two component areas A_1 and A_2. Determine the first moment of each component area with respect to the x axis, and explain the results obtained.

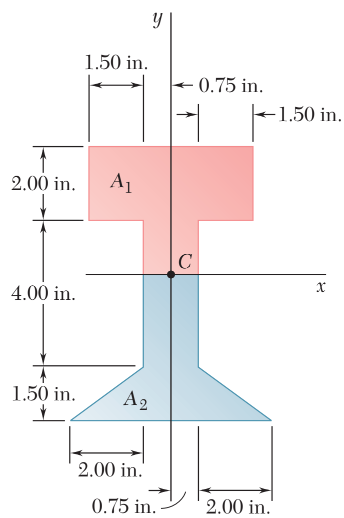


Fig. P5.22

5.23 The first moment of the shaded area with respect to the x axis is denoted by Q_x. (*a*) Express Q_x in terms of b, c, and the distance y from the base of the shaded area to the x axis. (*b*) For what value of y is Q_x maximum, and what is that maximum value?

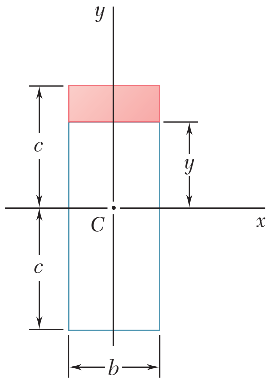


Fig. P5.23

5.24 through 5.27 A thin, homogeneous wire is bent to form the perimeter of the figure indicated. Locate the center of gravity of the wire figure thus formed.

5.24 Fig. P5.1.
5.25 Fig. P5.2.
5.26 Fig. P5.3.
5.27 Fig. P5.7.

5.28 A uniform circular rod of weight 8 lb and radius 10 in. is attached to a pin at C and to the cable AB. Determine (*a*) the tension in the cable, (*b*) the reaction at C.

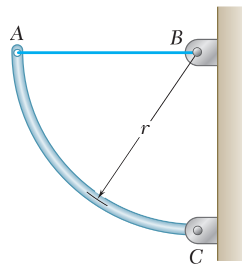


Fig. P5.28

5.29 Member $ABCDE$ is a component of a mobile and is formed from a single piece of aluminum tubing. Knowing that the member is supported at C and that $l = 2$ m, determine the distance d so that portion BCD of the member is horizontal.

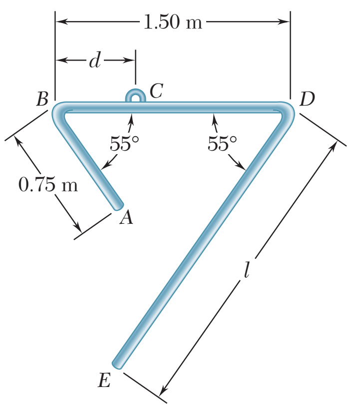


Fig. P5.29 and P5.30

5.30 Member $ABCDE$ is a component of a mobile and is formed from a single piece of aluminum tubing. Knowing that the member is supported at C and that d is 0.50 m, determine the length l of arm DE so that this portion of the member is horizontal.

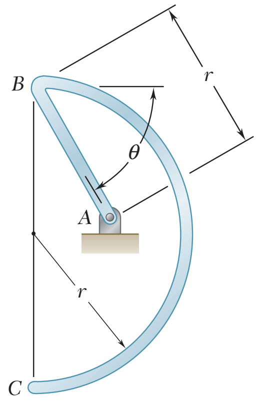


Fig. *P5.31*

5.31 The homogeneous wire ABC is bent into a semicircular arc and a straight section as shown and is attached to a hinge at A. Determine the value of θ for which the wire is in equilibrium for the indicated position.

5.32 Determine the distance h for which the centroid of the shaded area is as far above line BB' as possible when (*a*) $k = 0.10$, (*b*) $k = 0.80$.

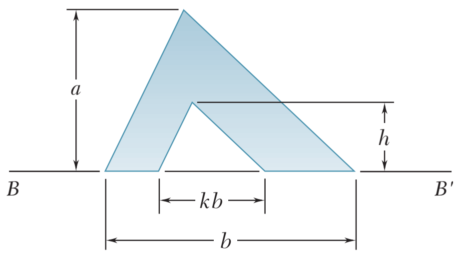


Fig. P5.32 and P5.33

5.33 Knowing that the distance h has been selected to maximize the distance $\bar{y}$ from line BB' to the centroid of the shaded area, show that $\bar{y} = 2h/3$.

5.6 DETERMINATION OF CENTROIDS BY INTEGRATION

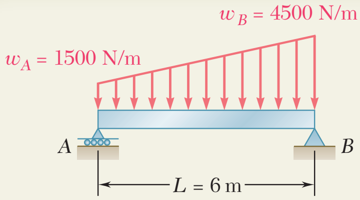
The centroid of an area bounded by analytical curves (i.e., curves defined by algebraic equations) is usually determined by evaluating the integrals in Eqs. (5.3) of Sec. 5.3:

$$\bar{x}A = \int x\, dA \qquad \bar{y}A = \int y\, dA \tag{5.3}$$

If the element of area dA is a small rectangle of sides dx and dy, the evaluation of each of these integrals requires a *double integration* with respect to x and y. A double integration is also necessary if polar coordinates are used for which dA is a small element of sides dr and $r\, d\theta$.

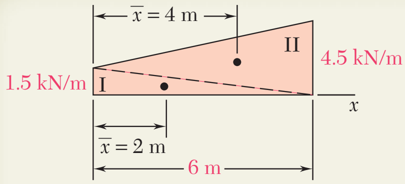
In most cases, however, it is possible to determine the coordinates of the centroid of an area by performing a single integration. This is achieved by choosing dA to be a thin rectangle or strip or a thin sector or pie-shaped element (Fig. 5.12); the centroid of the thin rectangle is located at its center, and the centroid of the thin sector is located at a distance $\frac{2}{3}r$ from its vertex (as it is for a triangle). The coordinates of the centroid of the area under consideration are then obtained by expressing that the first moment of the entire area with respect to each of the coordinate axes is equal to the sum (or integral) of the corresponding moments of the elements of area.

SAMPLE PROBLEM 5.9



A beam supports a distributed load as shown. (*a*) Determine the equivalent concentrated load. (*b*) Determine the reactions at the supports.

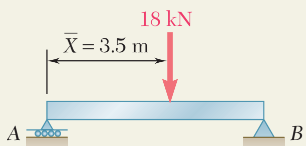
SOLUTION



a. Equivalent Concentrated Load. The magnitude of the resultant of the load is equal to the area under the load curve, and the line of action of the resultant passes through the centroid of the same area. We divide the area under the load curve into two triangles and construct the table below. To simplify the computations and tabulation, the given loads per unit length have been converted into kN/m.

Component	A, kN	$\bar{x}$, m	$\bar{x}A$, kN · m
Triangle I	4.5	2	9
Triangle II	13.5	4	54
	$\Sigma A = 18.0$		$\Sigma \bar{x}A = 63$

Thus, $\bar{X}\Sigma A = \Sigma \bar{x}A$: $\quad \bar{X}(18\text{ kN}) = 63\text{ kN} \cdot \text{m} \qquad \bar{X} = 3.5\text{ m}$



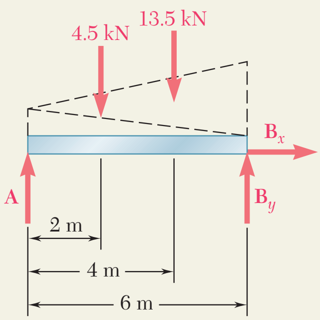
The equivalent concentrated load is

$$\mathbf{W} = 18\text{ kN}\downarrow \quad ◀$$

and its line of action is located at a distance

$$\bar{X} = 3.5\text{ m to the right of } A \quad ◀$$

b. Reactions. The reaction at A is vertical and is denoted by **A**; the reaction at B is represented by its components $\mathbf{B}_x$ and $\mathbf{B}_y$. The given load can be considered to be the sum of two triangular loads as shown. The resultant of each triangular load is equal to the area of the triangle and acts at its centroid. We write the following equilibrium equations for the free body shown:



$\overset{+}{\rightarrow}\Sigma F_x = 0$: $\qquad \mathbf{B}_x = 0 \quad ◀$

$+\circlearrowleft \Sigma M_A = 0$: $\qquad -(4.5\text{ kN})(2\text{ m}) - (13.5\text{ kN})(4\text{ m}) + B_y(6\text{ m}) = 0$

$$\mathbf{B}_y = 10.5\text{ kN}\uparrow \quad ◀$$

$+\circlearrowleft \Sigma M_B = 0$: $\qquad +(4.5\text{ kN})(4\text{ m}) + (13.5\text{ kN})(2\text{ m}) - A(6\text{ m}) = 0$

$$\mathbf{A} = 7.5\text{ kN}\uparrow \quad ◀$$

Alternative Solution. The given distributed load can be replaced by its resultant, which was found in part *a*. The reactions can be determined by writing the equilibrium equations $\Sigma F_x = 0$, $\Sigma M_A = 0$, and $\Sigma M_B = 0$. We again obtain

$$\mathbf{B}_x = 0 \qquad \mathbf{B}_y = 10.5\text{ kN}\uparrow \qquad \mathbf{A} = 7.5\text{ kN}\uparrow \quad ◀$$

PROBLEMS

5.66 and 5.67 For the beam and loading shown, determine (*a*) the magnitude and location of the resultant of the distributed load, (*b*) the reactions at the beam supports.

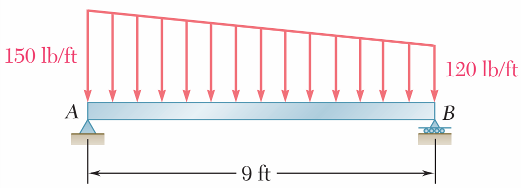


Fig. P5.66

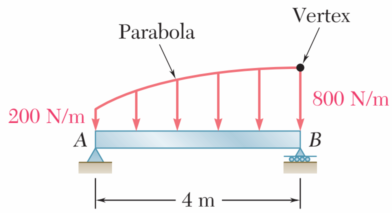


Fig. P5.67

5.68 through 5.73 Determine the reactions at the beam supports for the given loading.

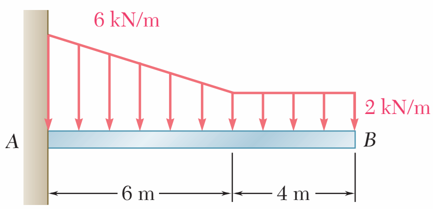


Fig. P5.68

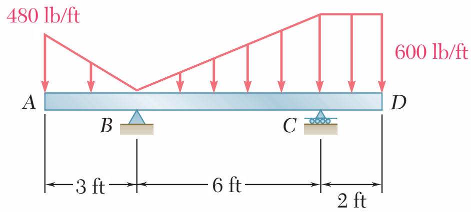


Fig. P5.69

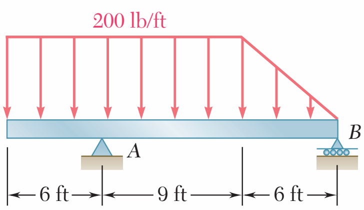


Fig. *P5.70*

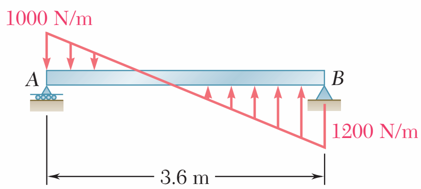


Fig. P5.71

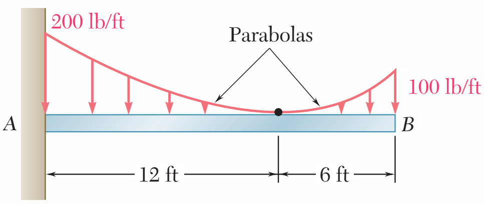


Fig. *P5.72*

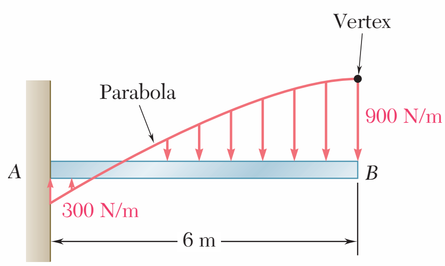


Fig. P5.73

5.74 Determine (*a*) the distance a so that the vertical reactions at supports A and B are equal, (*b*) the corresponding reactions at the supports.

5.75 Determine (*a*) the distance a so that the reaction at support B is minimum, (*b*) the corresponding reactions at the supports.

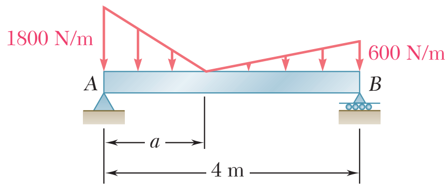


Fig. P5.74 and *P5.75*

5.76 Determine the reactions at the beam supports for the given loading when $w_0 = 150$ lb/ft.

5.77 Determine (*a*) the distributed load w_0 at the end D of the beam $ABCD$ for which the reaction at B is zero, (*b*) the corresponding reaction at C.

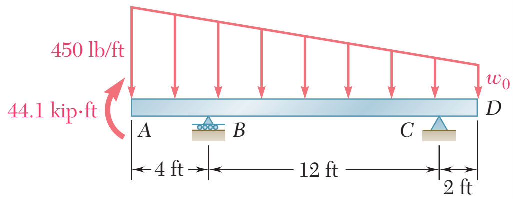


Fig. P5.76 and P5.77

5.78 The beam AB supports two concentrated loads and rests on soil that exerts a linearly distributed upward load as shown. Determine the values of w_A and w_B corresponding to equilibrium.

5.79 For the beam and loading of Prob. 5.78, determine (*a*) the distance a for which $w_A = 20$ kN/m, (*b*) the corresponding value of w_B.

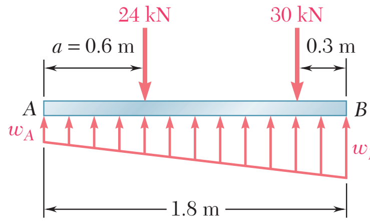


Fig. P5.78

In the following problems, use $\gamma = 62.4$ lb/ft^3 for the specific weight of fresh water and $\gamma_c = 150$ lb/ft^3 for the specific weight of concrete if U.S. customary units are used. With SI units, use $\rho = 10^3$ kg/m^3 for the density of fresh water and $\rho_c = 2.40 \times 10^3$ kg/m^3 for the density of concrete. (See the footnote on page 222 for how to determine the specific weight of a material given its density.)

5.80 The cross section of a concrete dam is as shown. For a 1-m-wide dam section determine (*a*) the resultant of the reaction forces exerted by the ground on the base AB of the dam, (*b*) the point of application of the resultant of part *a*, (*c*) the resultant of the pressure forces exerted by the water on the face BC of the dam.

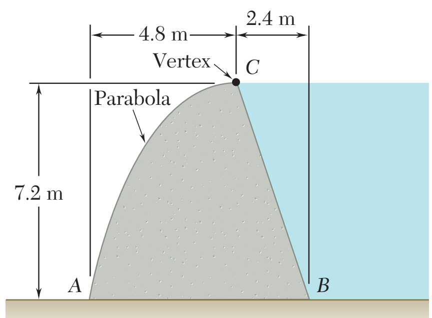


Fig. P5.80

5.81 The cross section of a concrete dam is as shown. For a 1-ft-wide dam section determine (*a*) the resultant of the reaction forces exerted by the ground on the base AB of the dam, (*b*) the point of application of the resultant of part *a*, (*c*) the resultant of the pressure forces exerted by the water on the face BC of the dam.

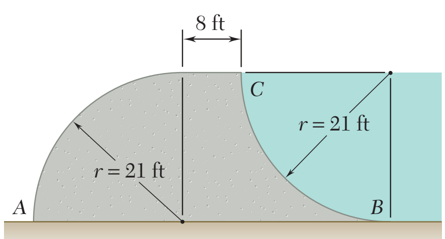
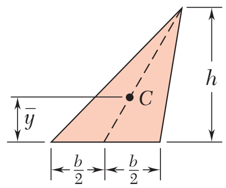
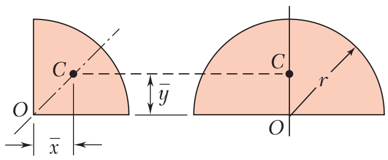
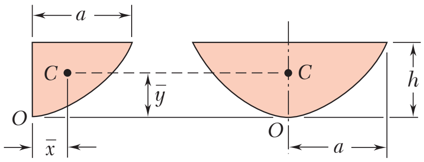
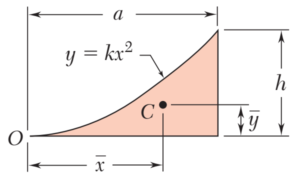
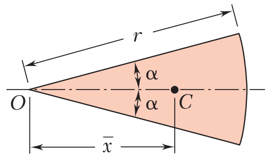
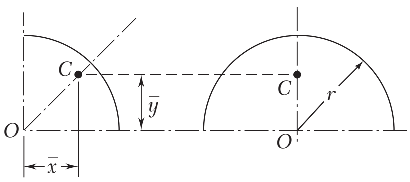
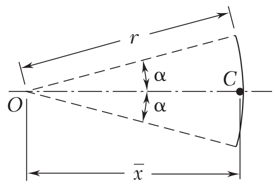


Fig. P5.81

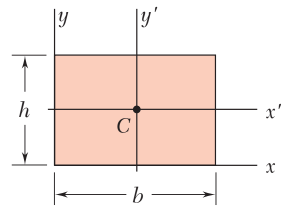
Centroids of Common Shapes of Areas and Lines

Shape		$\bar{x}$	$\bar{y}$	Area
Triangular area			$\frac{h}{3}$	$\frac{bh}{2}$
Quarter-circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
Semicircular area		0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$
Semiparabolic area		$\frac{3a}{8}$	$\frac{3h}{5}$	$\frac{2ah}{3}$
Parabolic area		0	$\frac{3h}{5}$	$\frac{4ah}{3}$
Parabolic spandrel		$\frac{3a}{4}$	$\frac{3h}{10}$	$\frac{ah}{3}$
Circular sector		$\frac{2r \sin\alpha}{3\alpha}$	0	αr^2
Quarter-circular arc		$\frac{2r}{\pi}$	$\frac{2r}{\pi}$	$\frac{\pi r}{2}$
Semicircular arc		0	$\frac{2r}{\pi}$	πr
Arc of circle		$\frac{r \sin\alpha}{\alpha}$	0	$2\alpha r$

Moments of Inertia of Common Geometric Shapes

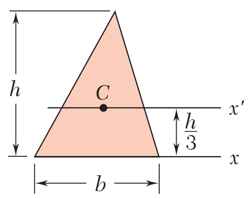
Rectangle

$\bar{I}_{x'} = \frac{1}{12}bh^3$
$\bar{I}_{y'} = \frac{1}{12}b^3h$
$I_x = \frac{1}{3}bh^3$
$I_y = \frac{1}{3}b^3h$
$J_C = \frac{1}{12}bh(b^2 + h^2)$



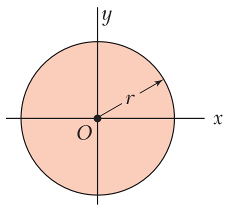
Triangle

$\bar{I}_{x'} = \frac{1}{36}bh^3$
$I_x = \frac{1}{12}bh^3$



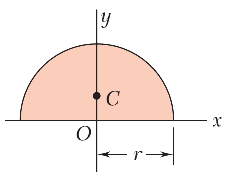
Circle

$\bar{I}_x = \bar{I}_y = \frac{1}{4}\pi r^4$
$J_O = \frac{1}{2}\pi r^4$



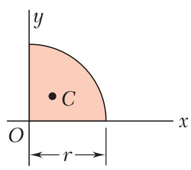
Semicircle

$I_x = I_y = \frac{1}{8}\pi r^4$
$J_O = \frac{1}{4}\pi r^4$



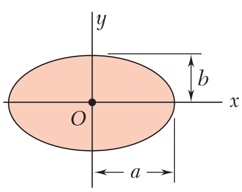
Quarter circle

$I_x = I_y = \frac{1}{16}\pi r^4$
$J_O = \frac{1}{8}\pi r^4$



Ellipse

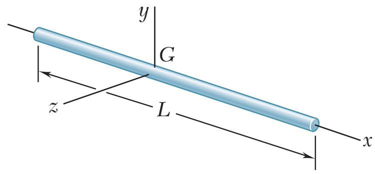
$\bar{I}_x = \frac{1}{4}\pi ab^3$
$\bar{I}_y = \frac{1}{4}\pi a^3b$
$J_O = \frac{1}{4}\pi ab(a^2 + b^2)$



Mass Moments of Inertia of Common Geometric Shapes

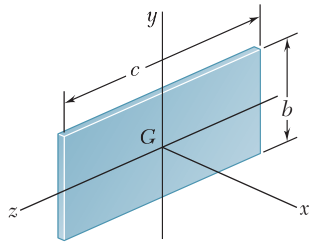
Slender rod

$I_y = I_z = \frac{1}{12}mL^2$



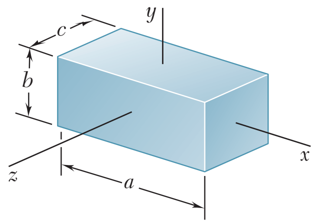
Thin rectangular plate

$I_x = \frac{1}{12}m(b^2 + c^2)$
$I_y = \frac{1}{12}mc^2$
$I_z = \frac{1}{12}mb^2$



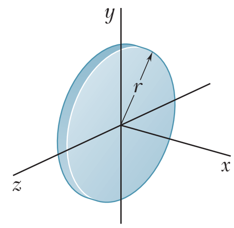
Rectangular prism

$I_x = \frac{1}{12}m(b^2 + c^2)$
$I_y = \frac{1}{12}m(c^2 + a^2)$
$I_z = \frac{1}{12}m(a^2 + b^2)$



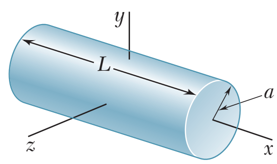
Thin disk

$I_x = \frac{1}{2}mr^2$
$I_y = I_z = \frac{1}{4}mr^2$



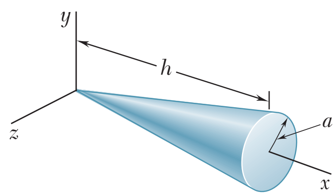
Circular cylinder

$I_x = \frac{1}{2}ma^2$
$I_y = I_z = \frac{1}{12}m(3a^2 + L^2)$



Circular cone

$I_x = \frac{3}{10}ma^2$
$I_y = I_z = \frac{3}{5}m(\frac{1}{4}a^2 + h^2)$



Sphere

$I_x = I_y = I_z = \frac{2}{5}ma^2$