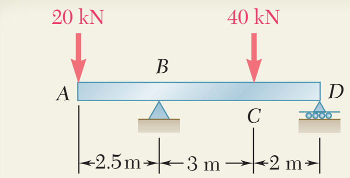
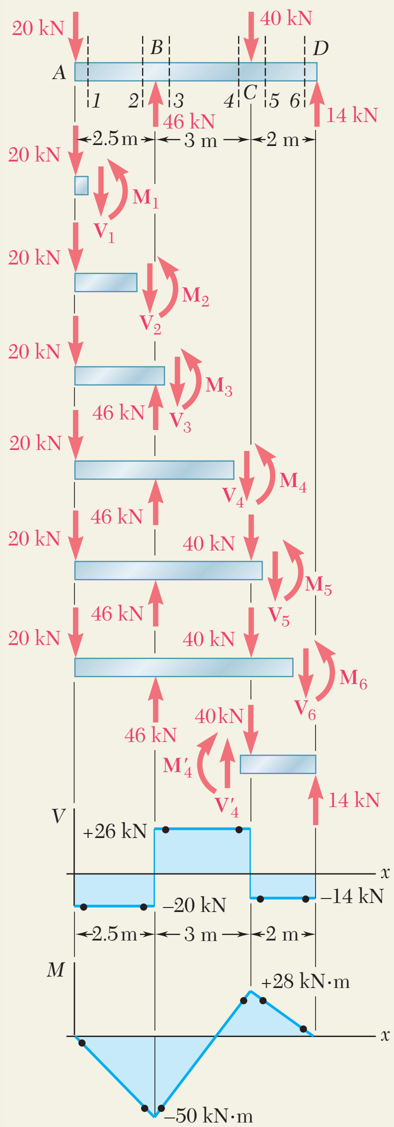


SAMPLE PROBLEM 7.2

Draw the shear and bending-moment diagrams for the beam and loading shown.



SOLUTION

Free-Body: Entire Beam. From the free-body diagram of the entire beam, we find the reactions at B and D:

$$\mathbf{R}_B = 46 \text{ kN} \uparrow \qquad \mathbf{R}_D = 14 \text{ kN} \uparrow$$

Shear and Bending Moment. We first determine the internal forces just to the right of the 20-kN load at A. Considering the stub of beam to the left of section *1* as a free body and assuming V and M to be positive (according to the standard convention), we write

$$+\uparrow \Sigma F_y = 0: \quad -20 \text{ kN} - V_1 = 0 \qquad V_1 = -20 \text{ kN}$$

$$+\circlearrowleft \Sigma M_1 = 0: \quad (20 \text{ kN})(0 \text{ m}) + M_1 = 0 \qquad M_1 = 0$$

We next consider as a free body the portion of the beam to the left of section 2 and write

$$+\uparrow \Sigma F_y = 0: \quad -20 \text{ kN} - V_2 = 0 \qquad V_2 = -20 \text{ kN}$$

$$+\circlearrowleft \Sigma M_2 = 0: \quad (20 \text{ kN})(2.5 \text{ m}) + M_2 = 0 \qquad M_2 = -50 \text{ kN} \cdot \text{m}$$

The shear and bending moment at sections *3*, *4*, *5*, and *6* are determined in a similar way from the free-body diagrams shown. We obtain

$$V_3 = +26 \text{ kN} \qquad M_3 = -50 \text{ kN} \cdot \text{m}$$
$$V_4 = +26 \text{ kN} \qquad M_4 = +28 \text{ kN} \cdot \text{m}$$
$$V_5 = -14 \text{ kN} \qquad M_5 = +28 \text{ kN} \cdot \text{m}$$
$$V_6 = -14 \text{ kN} \qquad M_6 = 0$$

For several of the latter sections, the results are more easily obtained by considering as a free body the portion of the beam to the right of the section. For example, considering the portion of the beam to the right of section *4*, we write

$$+\uparrow \Sigma F_y = 0: \quad V_4 - 40 \text{ kN} + 14 \text{ kN} = 0 \qquad V_4 = +26 \text{ kN}$$

$$+\circlearrowleft \Sigma M_4 = 0: \quad -M_4 + (14 \text{ kN})(2 \text{ m}) = 0 \qquad M_4 = +28 \text{ kN} \cdot \text{m}$$

Shear and Bending-Moment Diagrams. We can now plot the six points shown on the shear and bending-moment diagrams. As indicated in Sec. 7.5, the shear is of constant value between concentrated loads, and the bending moment varies linearly; we therefore obtain the shear and bending-moment diagrams shown.

PROBLEMS

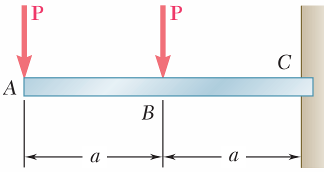


Fig. P7.29

7.29 through 7.32 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

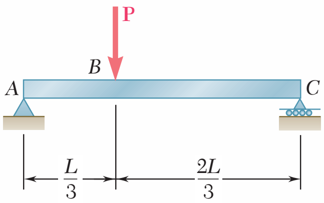


Fig. P7.30

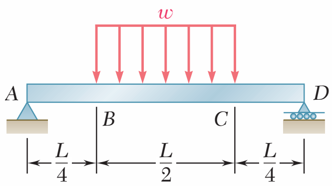


Fig. P7.31

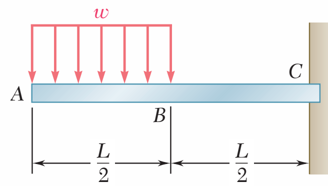


Fig. P7.32

7.33 and 7.34 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

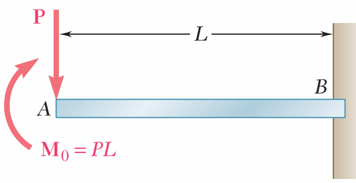


Fig. *P7.33*

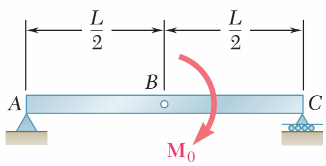


Fig. *P7.34*

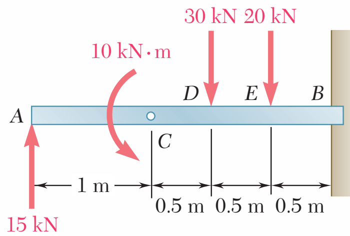


Fig. P7.35

7.35 and 7.36 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

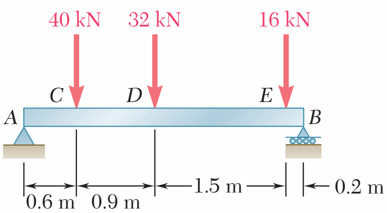


Fig. P7.36

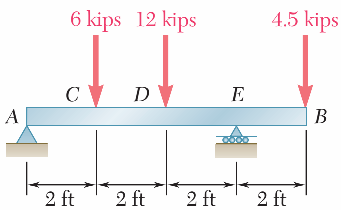


Fig. *P7.37*

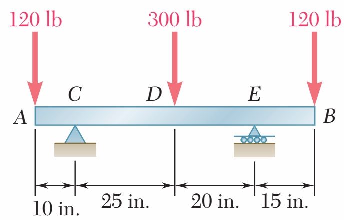


Fig. *P7.38*

7.37 and 7.38 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

7.39 through 7.42 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

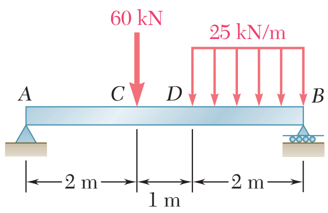


Fig. P7.39

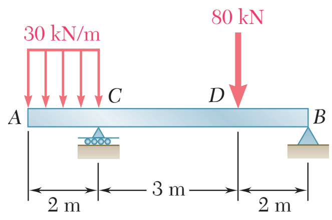


Fig. P7.40

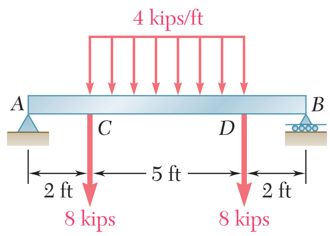


Fig. P7.41

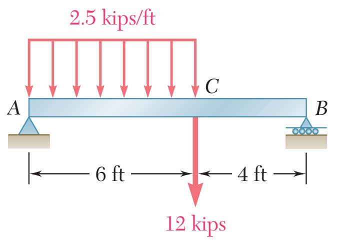


Fig. P7.42

7.43 Assuming the upward reaction of the ground on beam *AB* to be uniformly distributed and knowing that $a = 0.3$ m, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

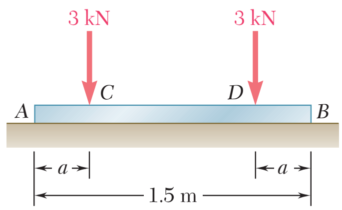


Fig. P7.43

7.44 Solve Prob. 7.43 knowing that $a = 0.5$ m.

7.45 and 7.46 Assuming the upward reaction of the ground on beam *AB* to be uniformly distributed, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

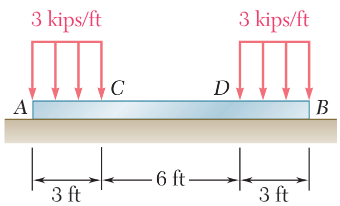


Fig. P7.45

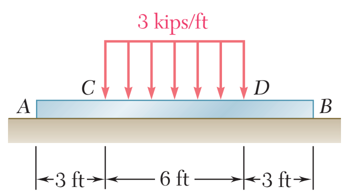


Fig. P7.46

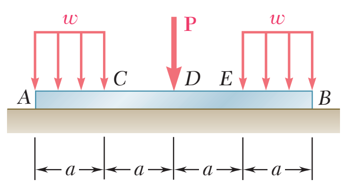


Fig. P7.47

7.47 Assuming the upward reaction of the ground on beam *AB* to be uniformly distributed and knowing that $P = wa$, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

7.48 Solve Prob. 7.47 knowing that $P = 3wa$.

7.49 Draw the shear and bending-moment diagrams for the beam *AB*, and determine the shear and bending moment (*a*) just to the left of *C*, (*b*) just to the right of *C*.

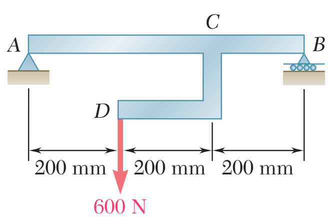


Fig. P7.49

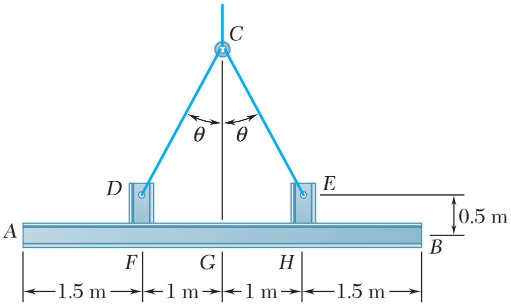


Fig. P7.50

7.50 Two small channel sections *DF* and *EH* have been welded to the uniform beam *AB* of weight $W = 3$ kN to form the rigid structural member shown. This member is being lifted by two cables attached at *D* and *E*. Knowing that $\theta = 30°$ and neglecting the weight of the channel sections, (*a*) draw the shear and bending-moment diagrams for beam *AB*, (*b*) determine the maximum absolute values of the shear and bending moment in the beam.

7.51 Solve Prob. 7.50 when $\theta = 60°$.

7.52 through 7.54 Draw the shear and bending-moment diagrams for the beam *AB*, and determine the maximum absolute values of the shear and bending moment.

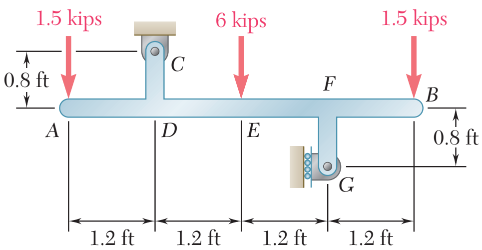


Fig. P7.52

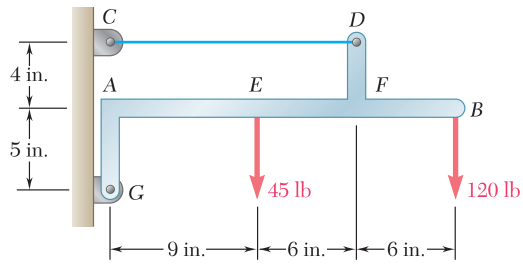


Fig. P7.53

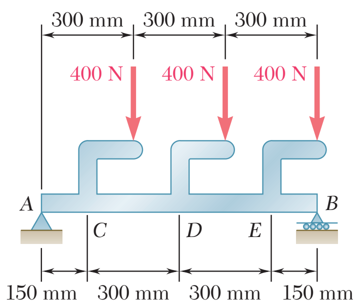


Fig. P7.54

7.55 For the structural member of Prob. 7.50, determine (*a*) the angle θ for which the maximum absolute value of the bending moment in beam *AB* is as small as possible, (*b*) the corresponding value of $|M|_{max}$. (*Hint:* Draw the bending-moment diagram and then equate the absolute values of the largest positive and negative bending moments obtained.)

7.56 For the beam of Prob. 7.43, determine (*a*) the distance *a* for which the maximum absolute value of the bending moment in the beam is as small as possible, (*b*) the corresponding value of $|M|_{max}$. (See hint for Prob. 7.55.)

7.57 For the beam of Prob. 7.47, determine (*a*) the ratio $k = P/wa$ for which the maximum absolute value of the bending moment in the beam is as small as possible, (*b*) the corresponding value of $|M|_{max}$. (See hint for Prob. 7.55.)

7.58 A uniform beam is to be picked up by crane cables attached at *A* and *B*. Determine the distance *a* from the ends of the beam to the points where the cables should be attached if the maximum absolute value of the bending moment in the beam is to be as small as possible. (*Hint:* Draw the bending-moment diagram in terms of *a*, *L*, and the weight *w* per unit length, and then equate the absolute values of the largest positive and negative bending moments obtained.)

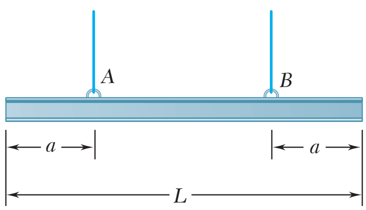


Fig. P7.58

7.59 For the beam shown, determine (*a*) the magnitude *P* of the two upward forces for which the maximum absolute value of the bending moment in the beam is as small as possible, (*b*) the corresponding value of $|M|_{max}$. (See hint for Prob. 7.55.)

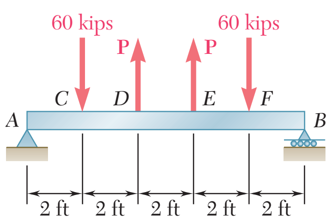
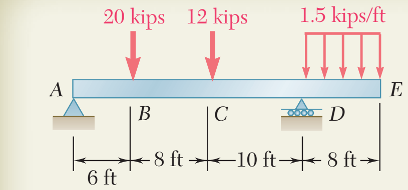


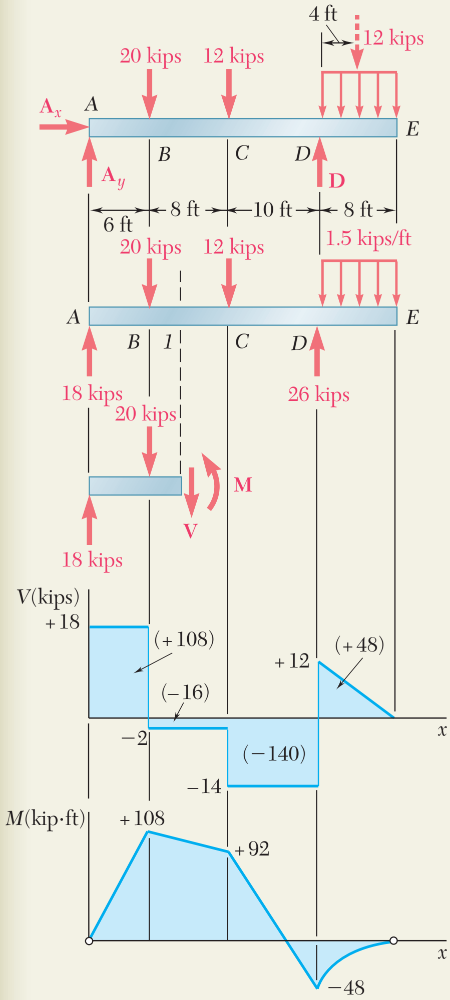
Fig. P7.59

SAMPLE PROBLEM 7.4



Draw the shear and bending-moment diagrams for the beam and loading shown.

SOLUTION



Free-Body: Entire Beam. Considering the entire beam as a free body, we determine the reactions:

$+\Sigma\circlearrowleft M_A = 0$:

$$D(24 \text{ ft}) - (20 \text{ kips})(6 \text{ ft}) - (12 \text{ kips})(14 \text{ ft}) - (12 \text{ kips})(28 \text{ ft}) = 0$$

$$D = +26 \text{ kips} \qquad \mathbf{D} = 26 \text{ kips} \uparrow$$

$+\uparrow\Sigma F_y = 0$: $\quad A_y - 20 \text{ kips} - 12 \text{ kips} + 26 \text{ kips} - 12 \text{ kips} = 0$

$$A_y = +18 \text{ kips} \qquad \mathbf{A}_y = 18 \text{ kips} \uparrow$$

$\overset{+}{\rightarrow}\Sigma F_x = 0$: $\quad A_x = 0 \qquad \mathbf{A}_x = 0$

We also note that at both A and E the bending moment is zero; thus two points (indicated by small circles) are obtained on the bending-moment diagram.

Shear Diagram. Since $dV/dx = -w$, we find that between concentrated loads and reactions the slope of the shear diagram is zero (i.e., the shear is constant). The shear at any point is determined by dividing the beam into two parts and considering either part as a free body. For example, using the portion of beam to the left of section *1*, we obtain the shear between B and C:

$+\uparrow\Sigma F_y = 0$: $\quad +18 \text{ kips} - 20 \text{ kips} - V = 0 \qquad V = -2 \text{ kips}$

We also find that the shear is +12 kips just to the right of D and zero at end E. Since the slope $dV/dx = -w$ is constant between D and E, the shear diagram between these two points is a straight line.

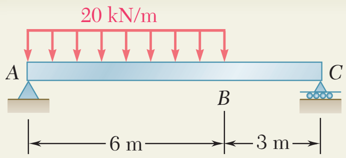
Bending-Moment Diagram. We recall that the area under the shear curve between two points is equal to the change in bending moment between the same two points. For convenience, the area of each portion of the shear diagram is computed and is indicated on the diagram. Since the bending moment M_A at the left end is known to be zero, we write

$$M_B - M_A = +108 \qquad M_B = +108 \text{ kip}\cdot\text{ft}$$
$$M_C - M_B = -\ 16 \qquad M_C = +\ 92 \text{ kip}\cdot\text{ft}$$
$$M_D - M_C = -140 \qquad M_D = -\ 48 \text{ kip}\cdot\text{ft}$$
$$M_E - M_D = +\ 48 \qquad M_E = 0$$

Since M_E is known to be zero, a check of the computations is obtained.

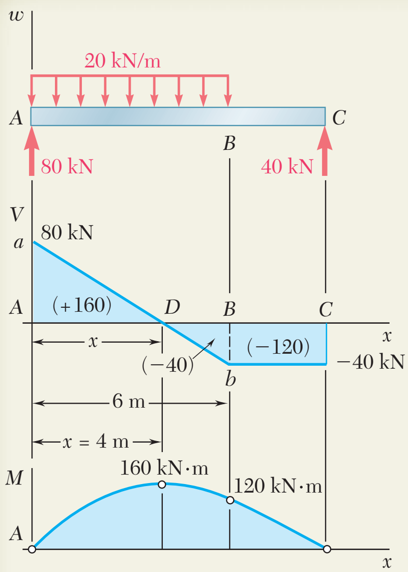
Between the concentrated loads and reactions the shear is constant; thus the slope dM/dx is constant, and the bending-moment diagram is drawn by connecting the known points with straight lines. Between D and E, where the shear diagram is an oblique straight line, the bending-moment diagram is a parabola.

From the V and M diagrams we note that $V_{max} = 18$ kips and $M_{max} = 108 \text{ kip}\cdot\text{ft}$.



SAMPLE PROBLEM 7.5

Draw the shear and bending-moment diagrams for the beam and loading shown and determine the location and magnitude of the maximum bending moment.



SOLUTION

Free-Body: Entire Beam. Considering the entire beam as a free body, we obtain the reactions

$$\mathbf{R}_A = 80 \text{ kN} \uparrow \qquad \mathbf{R}_C = 40 \text{ kN} \uparrow$$

Shear Diagram. The shear just to the right of A is $V_A = +80$ kN. Since the change in shear between two points is equal to *minus* the area under the load curve between the same two points, we obtain V_B by writing

$$V_B - V_A = -(20 \text{ kN/m})(6 \text{ m}) = -120 \text{ kN}$$
$$V_B = -120 + V_A = -120 + 80 = -40 \text{ kN}$$

Since the slope $dV/dx = -w$ is constant between A and B, the shear diagram between these two points is represented by a straight line. Between B and C, the area under the load curve is zero; therefore,

$$V_C - V_B = 0 \qquad V_C = V_B = -40 \text{ kN}$$

and the shear is constant between B and C.

Bending-Moment Diagram. We note that the bending moment at each end of the beam is zero. In order to determine the maximum bending moment, we locate the section D of the beam where $V = 0$. We write

$$V_D - V_A = -wx$$
$$0 - 80 \text{ kN} = -(20 \text{ kN/m})x$$

and, solving for x: $x = 4$ m ◀

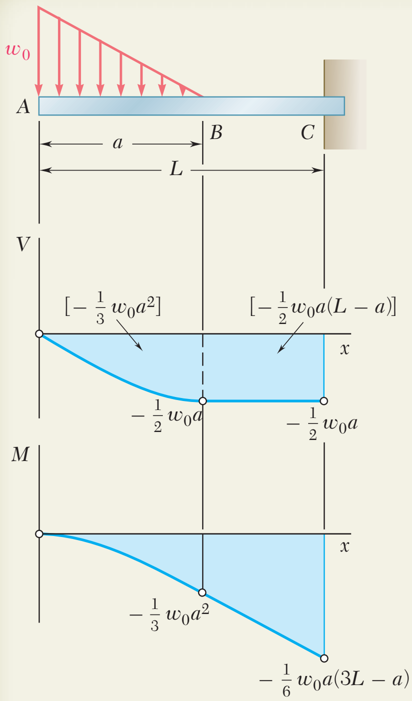
The maximum bending moment occurs at point D, where we have $dM/dx = V = 0$. The areas of the various portions of the shear diagram are computed and are given (in parentheses) on the diagram. Since the area of the shear diagram between two points is equal to the change in bending moment between the same two points, we write

$$M_D - M_A = +160 \text{ kN}\cdot\text{m} \qquad M_D = +160 \text{ kN}\cdot\text{m}$$
$$M_B - M_D = -\ 40 \text{ kN}\cdot\text{m} \qquad M_B = +120 \text{ kN}\cdot\text{m}$$
$$M_C - M_B = -120 \text{ kN}\cdot\text{m} \qquad M_C = 0$$

The bending-moment diagram consists of an arc of parabola followed by a segment of straight line; the slope of the parabola at A is equal to the value of V at that point.

The maximum bending moment is

$$M_{max} = M_D = +160 \text{ kN}\cdot\text{m} \quad ◀$$



SAMPLE PROBLEM 7.6

Sketch the shear and bending-moment diagrams for the cantilever beam shown.

SOLUTION

Shear Diagram. At the free end of the beam, we find $V_A = 0$. Between A and B, the area under the load curve is $\frac{1}{2}w_0a$; we find V_B by writing

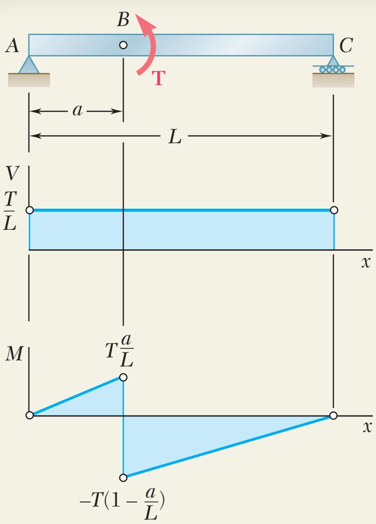
$$V_B - V_A = -\tfrac{1}{2}w_0a \qquad V_B = -\tfrac{1}{2}w_0a$$

Between B and C, the beam is not loaded; thus $V_C = V_B$. At A, we have $w = w_0$, and, according to Eq. (7.1), the slope of the shear curve is $dV/dx = -w_0$, while at B the slope is $dV/dx = 0$. Between A and B, the loading decreases linearly, and the shear diagram is parabolic. Between B and C, $w = 0$, and the shear diagram is a horizontal line.

Bending-Moment Diagram. We note that $M_A = 0$ at the free end of the beam. We compute the area under the shear curve and write

$$M_B - M_A = -\tfrac{1}{3}w_0a^2 \qquad M_B = -\tfrac{1}{3}w_0a^2$$
$$M_C - M_B = -\tfrac{1}{2}w_0a(L-a)$$
$$M_C = -\tfrac{1}{6}w_0a(3L-a)$$

The sketch of the bending-moment diagram is completed by recalling that $dM/dx = V$. We find that between A and B the diagram is represented by a cubic curve with zero slope at A, and between B and C the diagram is represented by a straight line.



SAMPLE PROBLEM 7.7

The simple beam AC is loaded by a couple of magnitude T applied at point B. Draw the shear and bending-moment diagrams for the beam.

SOLUTION

Free Body: Entire Beam. The entire beam is taken as a free body, and we obtain

$$\mathbf{R}_A = \frac{T}{L}\uparrow \qquad \mathbf{R}_C = \frac{T}{L}\downarrow$$

Shear and Bending-Moment Diagrams. The shear at any section is constant and equal to T/L. Since a couple is applied at B, the bending-moment diagram is discontinuous at B; the bending moment decreases suddenly by an amount equal to T.

PROBLEMS

7.63 Using the method of Sec. 7.6, solve Prob. 7.29.

7.64 Using the method of Sec. 7.6, solve Prob. 7.30.

7.65 Using the method of Sec. 7.6, solve Prob. 7.31.

7.66 Using the method of Sec. 7.6, solve Prob. 7.32.

7.67 Using the method of Sec. 7.6, solve Prob. 7.33.

7.68 Using the method of Sec. 7.6, solve Prob. 7.34.

7.69 and 7.70 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

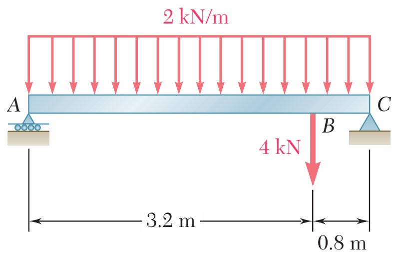


Fig. P7.69

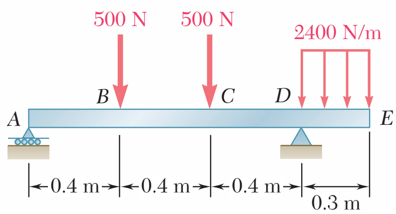


Fig. P7.70

7.71 Using the method of Sec. 7.6, solve Prob. 7.39.

7.72 Using the method of Sec. 7.6, solve Prob. 7.40.

7.73 Using the method of Sec. 7.6, solve Prob. 7.41.

7.74 Using the method of Sec. 7.6, solve Prob. 7.42.

7.75 and 7.76 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

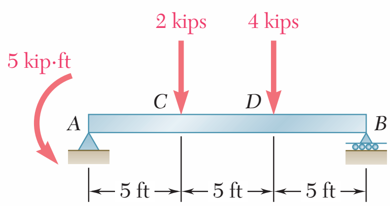


Fig. P7.75

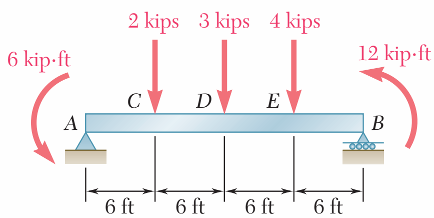


Fig. P7.76

7.77 and 7.78 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the magnitude and location of the maximum bending moment.

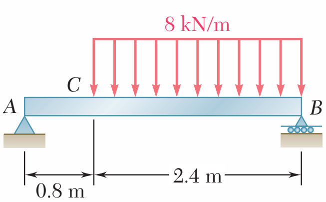


Fig. P7.77

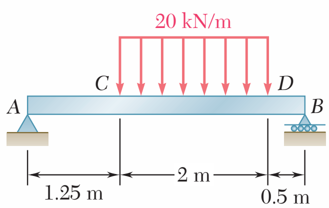


Fig. P7.78

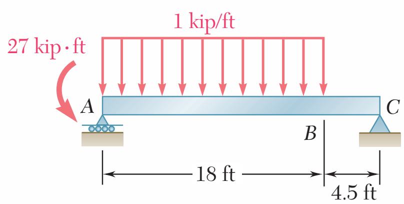


Fig. P7.79

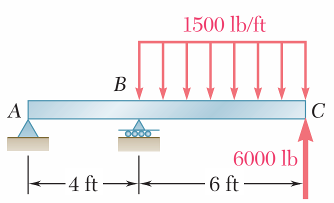


Fig. P7.80

7.79 and 7.80 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the magnitude and location of the maximum absolute value of the bending moment.

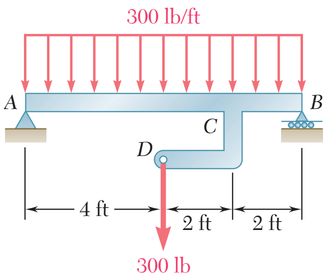


Fig. *P7.81*

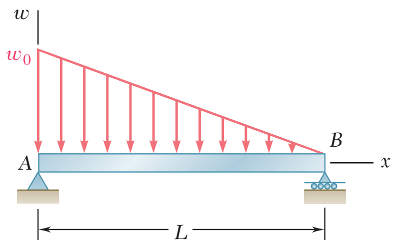


Fig. P7.85

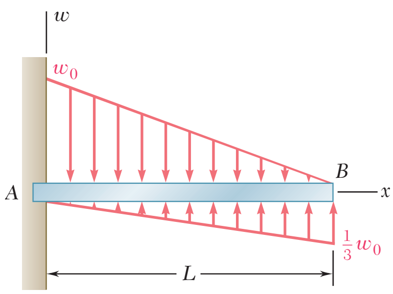


Fig. P7.86

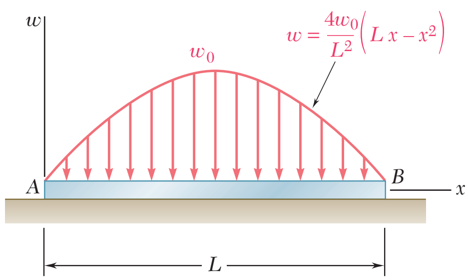


Fig. *P7.88*

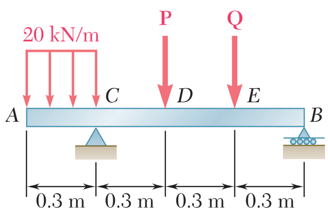


Fig. P7.89

7.81 (*a*) Draw the shear and bending-moment diagrams for beam *AB*, (*b*) determine the magnitude and location of the maximum absolute value of the bending moment.

7.82 Solve Prob. 7.81 assuming that the 300-lb force applied at *D* is directed upward.

7.83 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the magnitude and location of the maximum absolute value of the bending moment.

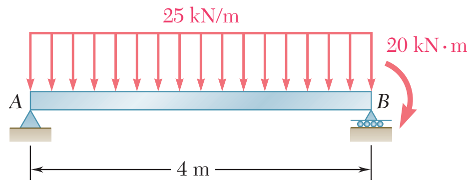


Fig. P7.83

7.84 Solve Prob. 7.83 assuming that the 20-kN · m couple applied at *B* is counterclockwise.

7.85 and 7.86 For the beam and loading shown, (*a*) write the equations of the shear and bending-moment curves, (*b*) determine the magnitude and location of the maximum bending moment.

7.87 For the beam and loading shown, (*a*) write the equations of the shear and bending-moment curves, (*b*) determine the magnitude and location of the maximum bending moment.

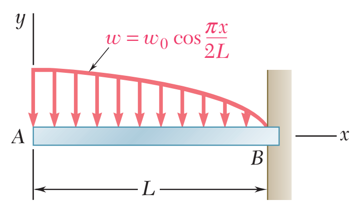


Fig. *P7.87*

7.88 The beam *AB*, which lies on the ground, supports the parabolic load shown. Assuming the upward reaction of the ground to be uniformly distributed, (*a*) write the equations of the shear and bending-moment curves, (*b*) determine the maximum bending moment.

7.89 The beam *AB* is subjected to the uniformly distributed load shown and to two unknown forces **P** and **Q**. Knowing that it has been experimentally determined that the bending moment is +800 N · m at *D* and +1300 N · m at *E*, (*a*) determine **P** and **Q**, (*b*) draw the shear and bending-moment diagrams for the beam.

7.90 Solve Prob. 7.89 assuming that the bending moment was found to be +650 N · m at *D* and +1450 N · m at *E*.