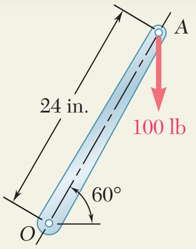
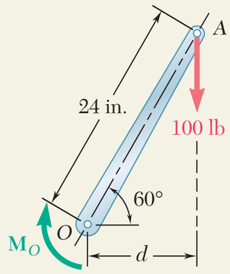
SAMPLE PROBLEM 3.1



A 100-lb vertical force is applied to the end of a lever which is attached to a shaft at O. Determine (*a*) the moment of the 100-lb force about O; (*b*) the horizontal force applied at A which creates the same moment about O; (*c*) the smallest force applied at A which creates the same moment about O; (*d*) how far from the shaft a 240-lb vertical force must act to create the same moment about O; (*e*) whether any one of the forces obtained in parts *b*, *c*, and *d* is equivalent to the original force.

SOLUTION



a. Moment about O. The perpendicular distance from O to the line of action of the 100-lb force is

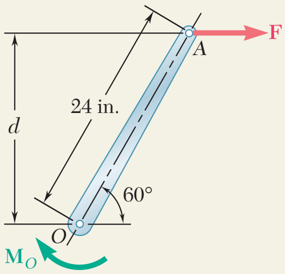
$$d = (24\text{ in.})\cos 60° = 12\text{ in.}$$

The magnitude of the moment about O of the 100-lb force is

$$M_O = Fd = (100\text{ lb})(12\text{ in.}) = 1200\text{ lb}\cdot\text{in.}$$

Since the force tends to rotate the lever clockwise about O, the moment will be represented by a vector $\mathbf{M}_O$ perpendicular to the plane of the figure and pointing *into* the paper. We express this fact by writing

$$\mathbf{M}_O = 1200\text{ lb}\cdot\text{in.}\ \circlearrowright$$ ◀

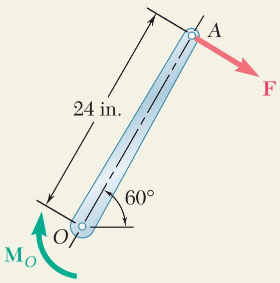


b. Horizontal Force. In this case, we have

$$d = (24\text{ in.})\sin 60° = 20.8\text{ in.}$$

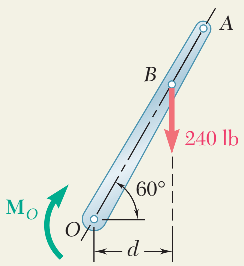
Since the moment about O must be 1200 lb · in., we write

$$M_O = Fd$$
$$1200\text{ lb}\cdot\text{in.} = F(20.8\text{ in.})$$
$$F = 57.7\text{ lb} \qquad \mathbf{F} = 57.7\text{ lb} \rightarrow$$ ◀



c. Smallest Force. Since $M_O = Fd$, the smallest value of F occurs when d is maximum. We choose the force perpendicular to OA and note that $d = 24$ in.; thus

$$M_O = Fd$$
$$1200\text{ lb}\cdot\text{in.} = F(24\text{ in.})$$
$$F = 50\text{ lb} \qquad \mathbf{F} = 50\text{ lb} \measuredangle 30°$$ ◀

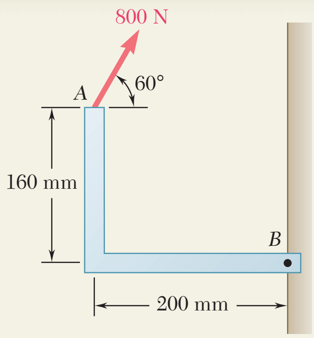


d. 240-lb Vertical Force. In this case $M_O = Fd$ yields

$$1200\text{ lb}\cdot\text{in.} = (240\text{ lb})d \qquad d = 5\text{ in.}$$

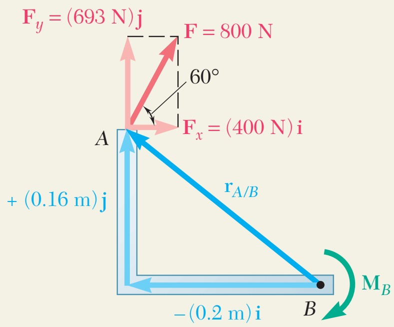
but $\quad OB\cos 60° = d \qquad OB = 10\text{ in.}$ ◀

e. None of the forces considered in parts *b*, *c*, and *d* is equivalent to the original 100-lb force. Although they have the same moment about O, they have different x and y components. In other words, although each force tends to rotate the shaft in the same manner, each causes the lever to pull on the shaft in a different way.



SAMPLE PROBLEM 3.2

A force of 800 N acts on a bracket as shown. Determine the moment of the force about B.



SOLUTION

The moment $\mathbf{M}_B$ of the force $\mathbf{F}$ about B is obtained by forming the vector product

$$\mathbf{M}_B = \mathbf{r}_{A/B} \times \mathbf{F}$$

where $\mathbf{r}_{A/B}$ is the vector drawn from B to A. Resolving $\mathbf{r}_{A/B}$ and $\mathbf{F}$ into rectangular components, we have

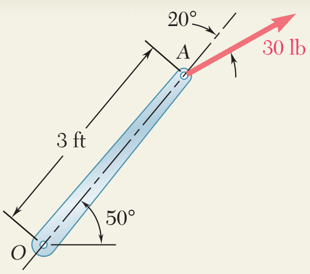
$$\begin{aligned}\mathbf{r}_{A/B} &= -(0.2\text{ m})\mathbf{i} + (0.16\text{ m})\mathbf{j}\\ \mathbf{F} &= (800\text{ N})\cos 60°\mathbf{i} + (800\text{ N})\sin 60°\mathbf{j}\\ &= (400\text{ N})\mathbf{i} + (693\text{ N})\mathbf{j}\end{aligned}$$

Recalling the relations (3.7) for the cross products of unit vectors (Sec. 3.5), we obtain

$$\begin{aligned}\mathbf{M}_B = \mathbf{r}_{A/B} \times \mathbf{F} &= [-(0.2\text{ m})\mathbf{i} + (0.16\text{ m})\mathbf{j}] \times [(400\text{ N})\mathbf{i} + (693\text{ N})\mathbf{j}]\\ &= -(138.6\text{ N}\cdot\text{m})\mathbf{k} - (64.0\text{ N}\cdot\text{m})\mathbf{k}\\ &= -(202.6\text{ N}\cdot\text{m})\mathbf{k}\end{aligned}$$

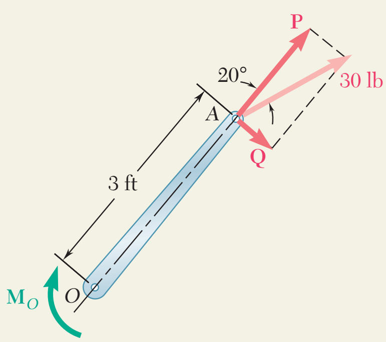
$$\mathbf{M}_B = 203\text{ N}\cdot\text{m} \ \circlearrowright \quad ◀$$

The moment $\mathbf{M}_B$ is a vector perpendicular to the plane of the figure and pointing *into* the paper.



SAMPLE PROBLEM 3.3

A 30-lb force acts on the end of the 3-ft lever as shown. Determine the moment of the force about O.



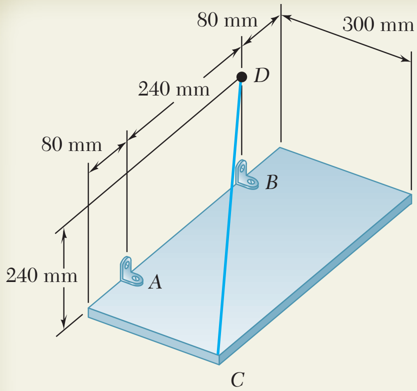
SOLUTION

The force is replaced by two components, one component $\mathbf{P}$ in the direction of OA and one component $\mathbf{Q}$ perpendicular to OA. Since O is on the line of action of $\mathbf{P}$, the moment of $\mathbf{P}$ about O is zero and the moment of the 30-lb force reduces to the moment of $\mathbf{Q}$, which is clockwise and, thus, is represented by a negative scalar.

$$Q = (30\text{ lb})\sin 20° = 10.26\text{ lb}$$
$$M_O = -Q(3\text{ ft}) = -(10.26\text{ lb})(3\text{ ft}) = -30.8\text{ lb}\cdot\text{ft}$$

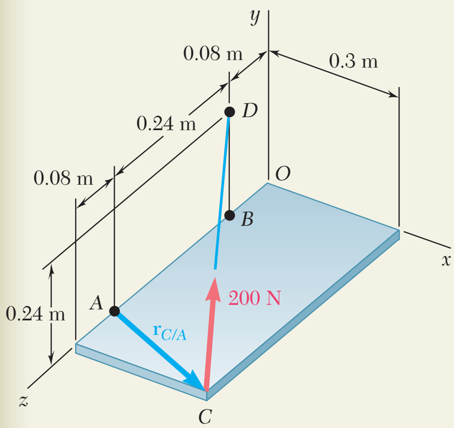
Since the value obtained for the scalar M_O is negative, the moment $\mathbf{M}_O$ points *into* the paper. We write

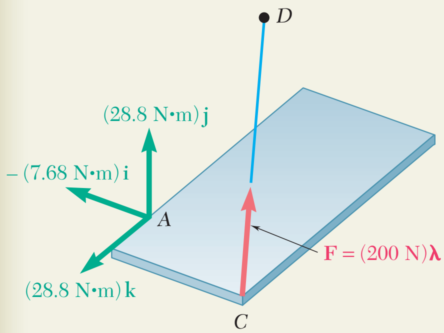
$$\mathbf{M}_O = 30.8\text{ lb}\cdot\text{ft} \ \circlearrowright \quad ◀$$



SAMPLE PROBLEM 3.4

A rectangular plate is supported by brackets at A and B and by a wire CD. Knowing that the tension in the wire is 200 N, determine the moment about A of the force exerted by the wire on point C.





SOLUTION

The moment $\mathbf{M}_A$ about A of the force $\mathbf{F}$ exerted by the wire on point C is obtained by forming the vector product

$$\mathbf{M}_A = \mathbf{r}_{C/A} \times \mathbf{F} \tag{1}$$

where $\mathbf{r}_{C/A}$ is the vector drawn from A to C,

$$\mathbf{r}_{C/A} = \overrightarrow{AC} = (0.3\text{ m})\mathbf{i} + (0.08\text{ m})\mathbf{k} \tag{2}$$

and $\mathbf{F}$ is the 200-N force directed along CD. Introducing the unit vector $\boldsymbol{\lambda} = \overrightarrow{CD}/CD$, we write

$$\mathbf{F} = F\boldsymbol{\lambda} = (200\text{ N})\frac{\overrightarrow{CD}}{CD} \tag{3}$$

Resolving the vector $\overrightarrow{CD}$ into rectangular components, we have

$$\overrightarrow{CD} = -(0.3\text{ m})\mathbf{i} + (0.24\text{ m})\mathbf{j} - (0.32\text{ m})\mathbf{k} \qquad CD = 0.50\text{ m}$$

Substituting into (3), we obtain

$$\mathbf{F} = \frac{200\text{ N}}{0.50\text{ m}}[-(0.3\text{ m})\mathbf{i} + (0.24\text{ m})\mathbf{j} - (0.32\text{ m})\mathbf{k}]$$

$$= -(120\text{ N})\mathbf{i} + (96\text{ N})\mathbf{j} - (128\text{ N})\mathbf{k} \tag{4}$$

Substituting for $\mathbf{r}_{C/A}$ and $\mathbf{F}$ from (2) and (4) into (1) and recalling the relations (3.7) of Sec. 3.5, we obtain

$$\mathbf{M}_A = \mathbf{r}_{C/A} \times \mathbf{F} = (0.3\mathbf{i} + 0.08\mathbf{k}) \times (-120\mathbf{i} + 96\mathbf{j} - 128\mathbf{k})$$

$$= (0.3)(96)\mathbf{k} + (0.3)(-128)(-\mathbf{j}) + (0.08)(-120)\mathbf{j} + (0.08)(96)(-\mathbf{i})$$

$$\mathbf{M}_A = -(7.68\text{ N}\cdot\text{m})\mathbf{i} + (28.8\text{ N}\cdot\text{m})\mathbf{j} + (28.8\text{ N}\cdot\text{m})\mathbf{k} \quad \blacktriangleleft$$

Alternative Solution. As indicated in Sec. 3.8, the moment $\mathbf{M}_A$ can be expressed in the form of a determinant:

$$\mathbf{M}_A = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x_C - x_A & y_C - y_A & z_C - z_A \\ F_x & F_y & F_z \end{vmatrix} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.3 & 0 & 0.08 \\ -120 & 96 & -128 \end{vmatrix}$$

$$\mathbf{M}_A = -(7.68\text{ N}\cdot\text{m})\mathbf{i} + (28.8\text{ N}\cdot\text{m})\mathbf{j} + (28.8\text{ N}\cdot\text{m})\mathbf{k} \quad \blacktriangleleft$$

SOLVING PROBLEMS ON YOUR OWN

In this lesson we introduced the *vector product* or *cross product* of two vectors. In the following problems, you may want to use the vector product to compute the *moment of a force about a point* and also to determine the *perpendicular distance* from a point to a line.

We defined the moment of the force **F** about the point O of a rigid body as

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} \tag{3.11}$$

where **r** is the position vector *from O to any point* on the line of action of **F**. Since the vector product is not commutative, it is absolutely necessary when computing such a product that you place the vectors in the proper order and that each vector have the correct sense. The moment $\mathbf{M}_O$ is important because its magnitude is a measure of the tendency of the force **F** to cause the rigid body to rotate about an axis directed along $\mathbf{M}_O$.

1. Computing the moment $\mathbf{M}_O$ of a force in two dimensions. You can use one of the following procedures:

a. Use Eq. (3.12), $M_O = Fd$, which expresses the magnitude of the moment as the product of the magnitude of **F** and the *perpendicular distance d* from O to the line of action of **F** (Sample Prob. 3.1).

b. Express **r** and **F** in component form and formally evaluate the vector product $\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$ [Sample Prob. 3.2].

c. Resolve **F** into components respectively parallel and perpendicular to the position vector **r**. Only the perpendicular component contributes to the moment of **F** [Sample Prob. 3.3].

d. Use Eq. (3.22), $M_O = M_z = xF_y - yF_x$. When applying this method, the simplest approach is to treat the scalar components of **r** and **F** as positive and then to assign, by observation, the proper sign to the moment produced by each force component. For example, applying this method to solve Sample Prob. 3.2, we observe that both force components tend to produce a clockwise rotation about B. Therefore, the moment of each force about B should be represented by a negative scalar. We then have for the total moment

$$M_B = -(0.16 \text{ m})(400 \text{ N}) - (0.20 \text{ m})(693 \text{ N}) = -202.6 \text{ N} \cdot \text{m}$$

2. Computing the moment $\mathbf{M}_O$ of a force F in three dimensions. Following the method of Sample Prob. 3.4, the first step in the process is to select the most convenient (simplest) position vector **r**. You should next express **F** in terms of its rectangular components. The final step is to evaluate the vector product $\mathbf{r} \times \mathbf{F}$ to determine the moment. In most three-dimensional problems you will find it easiest to calculate the vector product using a determinant.

3. Determining the perpendicular distance d from a point A to a given line. First assume that a force **F** of known magnitude F lies along the given line. Next determine its moment about A by forming the vector product $\mathbf{M}_A = \mathbf{r} \times \mathbf{F}$, and calculate this product as indicated above. Then compute its magnitude M_A. Finally, substitute the values of F and M_A into the equation $M_A = Fd$ and solve for d.

PROBLEMS

3.1 A foot valve for a pneumatic system is hinged at *B*. Knowing that $\alpha = 28°$, determine the moment of the 16-N force about point *B* by resolving the force into horizontal and vertical components.

3.2 A foot valve for a pneumatic system is hinged at *B*. Knowing that $\alpha = 28°$, determine the moment of the 16-N force about point *B* by resolving the force into components along *ABC* and in a direction perpendicular to *ABC*.

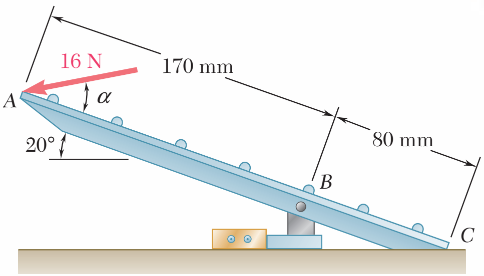


Fig. P3.1 and P3.2

3.3 A 300-N force is applied at *A* as shown. Determine (*a*) the moment of the 300-N force about *D*, (*b*) the smallest force applied at *B* that creates the same moment about *D*.

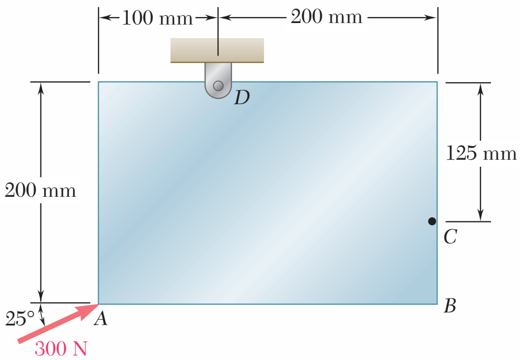


Fig. P3.3 and P3.4

3.4 A 300-N force is applied at *A* as shown. Determine (*a*) the moment of the 300-N force about *D*, (*b*) the magnitude and sense of the horizontal force applied at *C* that creates the same moment about *D*, (*c*) the smallest force applied at *C* that creates the same moment about *D*.

3.5 An 8-lb force **P** is applied to a shift lever. Determine the moment of **P** about *B* when α is equal to 25°.

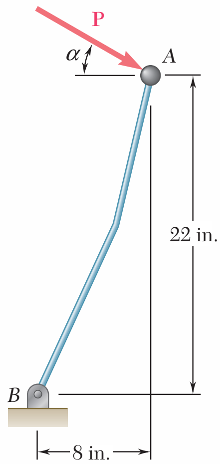


Fig. P3.5, *P3.6,* and P3.7

3.6 For the shift lever shown, determine the magnitude and the direction of the smallest force **P** that has a 210-lb · in. clockwise moment about *B*.

3.7 An 11-lb force **P** is applied to a shift lever. The moment of **P** about *B* is clockwise and has a magnitude of 250 lb · in. Determine the value of α.

3.8 It is known that a vertical force of 200 lb is required to remove the nail at *C* from the board. As the nail first starts moving, determine (*a*) the moment about *B* of the force exerted on the nail, (*b*) the magnitude of the force **P** that creates the same moment about *B* if $\alpha = 10°$, (*c*) the smallest force **P** that creates the same moment about *B*.

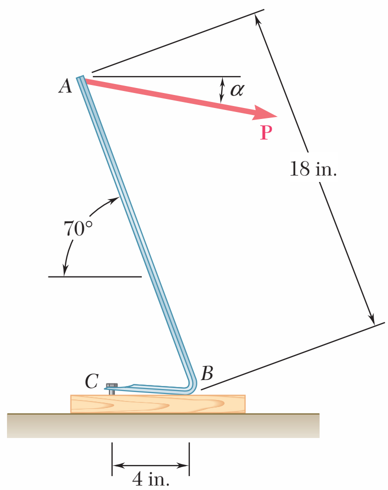


Fig. *P3.8*

3.9 A winch puller *AB* is used to straighten a fence post. Knowing that the tension in cable *BC* is 1040 N and length *d* is 1.90 m, determine the moment about *D* of the force exerted by the cable at *C* by resolving that force into horizontal and vertical components applied (*a*) at point *C*, (*b*) at point *E*.

3.10 It is known that a force with a moment of 960 N · m about *D* is required to straighten the fence post *CD*. If $d = 2.80$ m, determine the tension that must be developed in the cable of winch puller *AB* to create the required moment about point *D*.

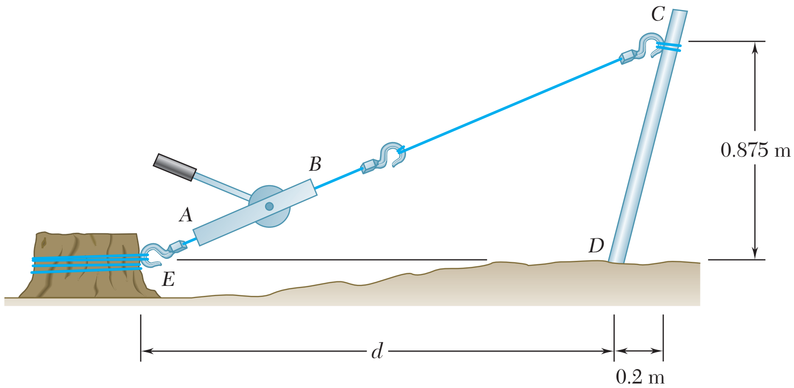


Fig. P3.9, P3.10, and *P3.11*

3.11 It is known that a force with a moment of 960 N · m about *D* is required to straighten the fence post *CD*. If the capacity of winch puller *AB* is 2400 N, determine the minimum value of distance *d* to create the specified moment about point *D*.

3.12 and 3.13 The tailgate of a car is supported by the hydraulic lift *BC*. If the lift exerts a 125-lb force directed along its centerline on the ball and socket at *B*, determine the moment of the force about *A*.

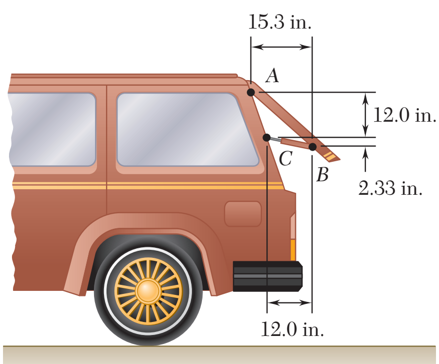


Fig. P3.12

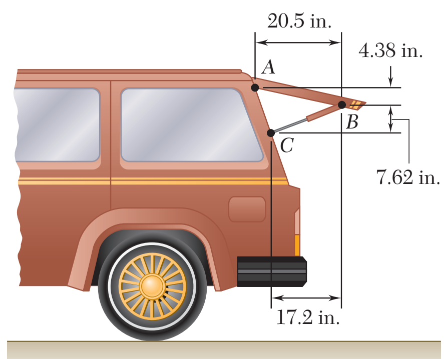


Fig. P3.13

3.14 A mechanic uses a piece of pipe AB as a lever when tightening an alternator belt. When he pushes down at A, a force of 485 N is exerted on the alternator at B. Determine the moment of that force about bolt C if its line of action passes through O.

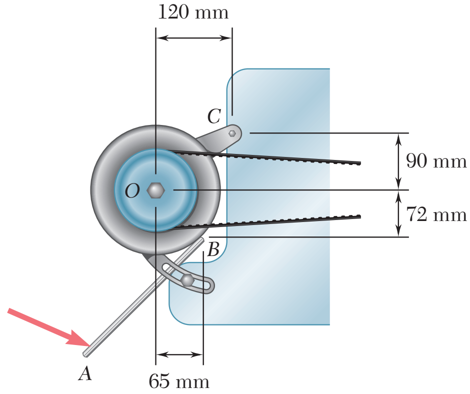


Fig. *P3.14*

3.15 Form the vector products $\mathbf{B} \times \mathbf{C}$ and $\mathbf{B}' \times \mathbf{C}$, where $B = B'$, and use the results obtained to prove the identity

$$\sin \alpha \cos \beta = \tfrac{1}{2} \sin (\alpha + \beta) + \tfrac{1}{2} \sin (\alpha - \beta).$$

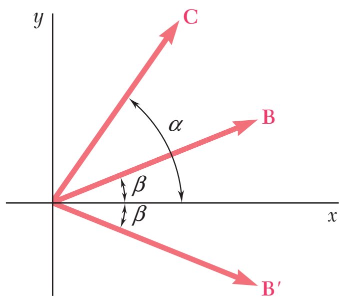


Fig. P3.15

3.16 A line passes through the points (20 m, 16 m) and (−1 m, −4 m). Determine the perpendicular distance d from the line to the origin O of the system of coordinates.

3.17 The vectors **P** and **Q** are two adjacent sides of a parallelogram. Determine the area of the parallelogram when (*a*) $\mathbf{P} = -7\mathbf{i} + 3\mathbf{j} - 3\mathbf{k}$ and $\mathbf{Q} = 2\mathbf{i} + 2\mathbf{j} + 5\mathbf{k}$, (*b*) $\mathbf{P} = 6\mathbf{i} - 5\mathbf{j} - 2\mathbf{k}$ and $\mathbf{Q} = -2\mathbf{i} + 5\mathbf{j} - \mathbf{k}$.

3.18 A plane contains the vectors **A** and **B**. Determine the unit vector normal to the plane when **A** and **B** are equal to, respectively, (*a*) $\mathbf{i} + 2\mathbf{j} - 5\mathbf{k}$ and $4\mathbf{i} - 7\mathbf{j} - 5\mathbf{k}$, (*b*) $3\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$ and $-2\mathbf{i} + 6\mathbf{j} - 4\mathbf{k}$.

3.19 Determine the moment about the origin O of the force $\mathbf{F} = 4\mathbf{i} + 5\mathbf{j} - 3\mathbf{k}$ that acts at a point A. Assume that the position vector of A is (*a*) $\mathbf{r} = 2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$, (*b*) $\mathbf{r} = 2\mathbf{i} + 2.5\mathbf{j} - 1.5\mathbf{k}$, (c) $\mathbf{r} = 2\mathbf{i} + 5\mathbf{j} + 6\mathbf{k}$.

3.20 Determine the moment about the origin O of the force $\mathbf{F} = -2\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}$ that acts at a point A. Assume that the position vector of A is (*a*) $\mathbf{r} = \mathbf{i} + \mathbf{j} + \mathbf{k}$, (*b*) $\mathbf{r} = 2\mathbf{i} + 3\mathbf{j} - 5\mathbf{k}$, (c) $\mathbf{r} = -4\mathbf{i} + 6\mathbf{j} + 10\mathbf{k}$.

3.21 A 200-N force is applied as shown to the bracket ABC. Determine the moment of the force about A.

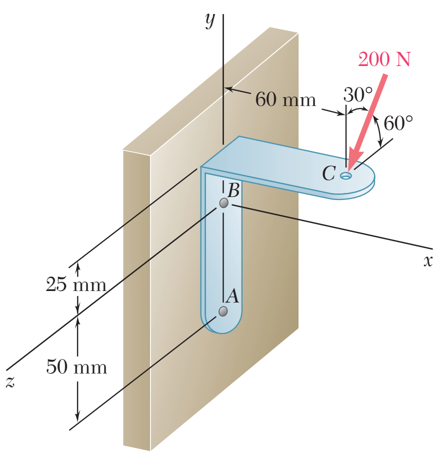


Fig. P3.21

3.22 Before the trunk of a large tree is felled, cables *AB* and *BC* are attached as shown. Knowing that the tensions in cables *AB* and *BC* are 555 N and 660 N, respectively, determine the moment about *O* of the resultant force exerted on the tree by the cables at *B*.

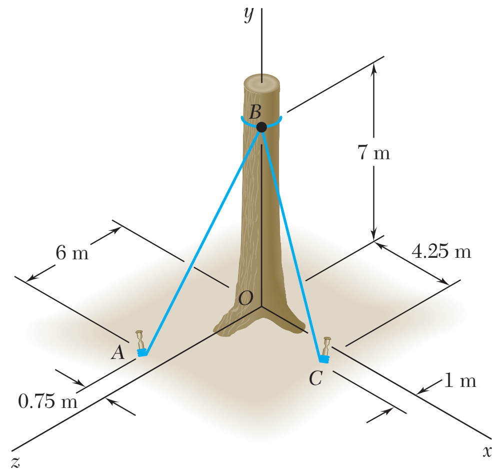


Fig. P3.22

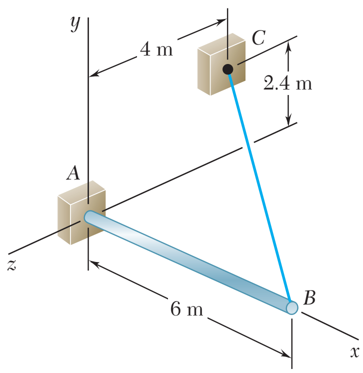


Fig. *P3.23*

3.23 The 6-m boom *AB* has a fixed end *A*. A steel cable is stretched from the free end *B* of the boom to a point *C* located on the vertical wall. If the tension in the cable is 2.5 kN, determine the moment about *A* of the force exerted by the cable at *B*.

3.24 A wooden board *AB*, which is used as a temporary prop to support a small roof, exerts at point *A* of the roof a 57-lb force directed along *BA*. Determine the moment about *C* of that force.

3.25 The ramp *ABCD* is supported by cables at corners *C* and *D*. The tension in each of the cables is 810 N. Determine the moment about *A* of the force exerted by (*a*) the cable at *D*, (*b*) the cable at *C*.

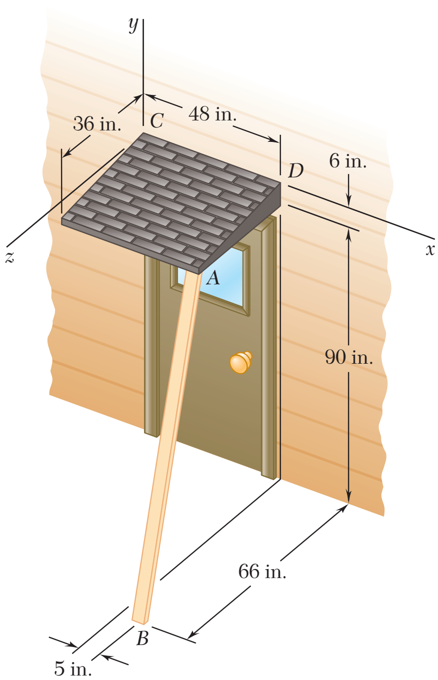


Fig. P3.24

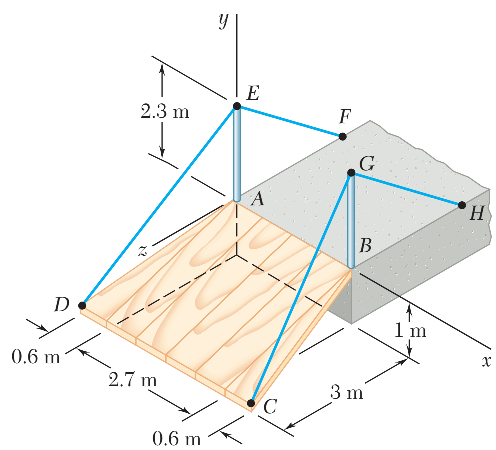


Fig. *P3.25*

3.26 A small boat hangs from two davits, one of which is shown in the figure. The tension in line *ABAD* is 82 lb. Determine the moment about *C* of the resultant force $\mathbf{R}_A$ exerted on the davit at *A*.

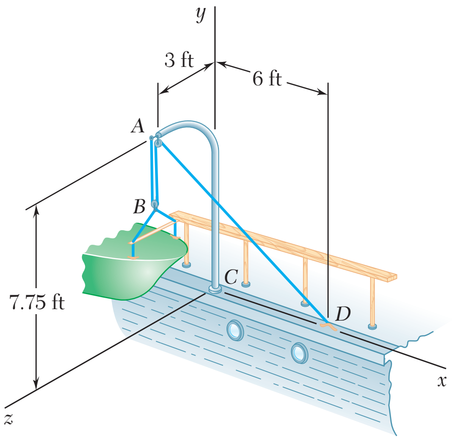


Fig. P3.26

3.27 In Prob. 3.22, determine the perpendicular distance from point *O* to cable *AB*.

3.28 In Prob. 3.22, determine the perpendicular distance from point *O* to cable *BC*.

3.29 In Prob. 3.24, determine the perpendicular distance from point *D* to a line drawn through points *A* and *B*.

3.30 In Prob. 3.24, determine the perpendicular distance from point *C* to a line drawn through points *A* and *B*.

3.31 In Prob. 3.25, determine the perpendicular distance from point *A* to portion *DE* of cable *DEF*.

3.32 In Prob. 3.25, determine the perpendicular distance from point *A* to a line drawn through points *C* and *G*.

3.33 In Prob. 3.26, determine the perpendicular distance from point *C* to portion *AD* of the line *ABAD*.

3.34 Determine the value of *a* that minimizes the perpendicular distance from point *C* to a section of pipeline that passes through points *A* and *B*.

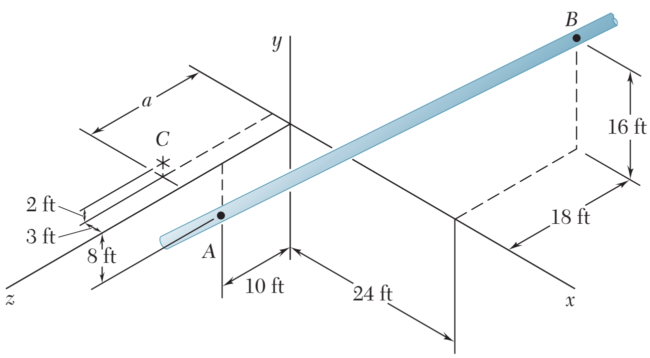


Fig. *P3.34*