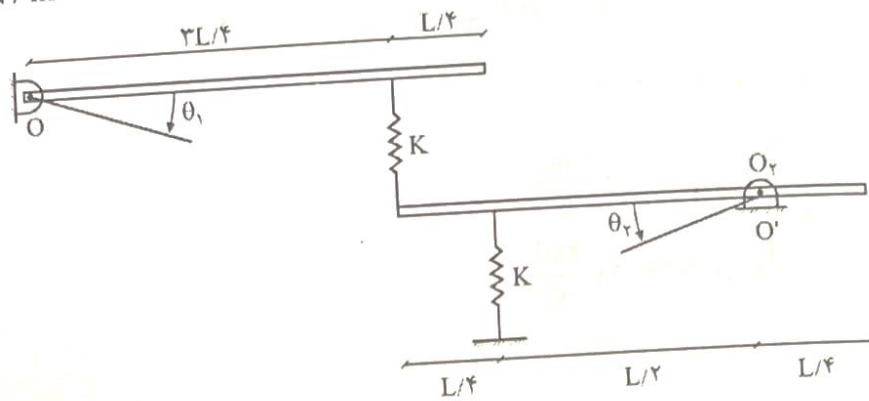


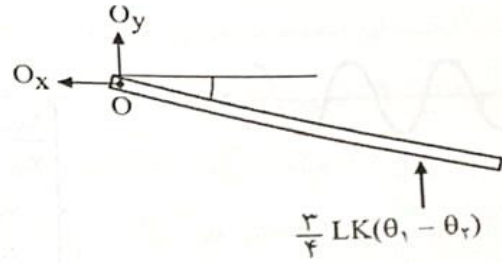
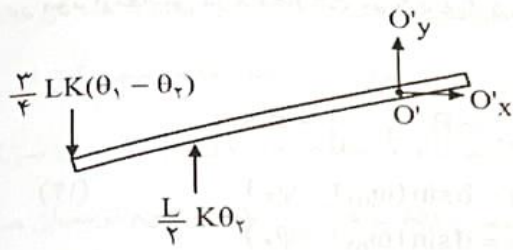
مثال

فرکانس‌های طبیعی و شکل مودهای ارتعاشی را در سیستم زیر بدست آورید. جرم و طول دو میله مساوی است.

$$m = 10 \text{ Kg}$$

$$K = 1000 \text{ N/m}$$





$$\begin{cases} \sum T_O = I_O \ddot{\theta}_1 \\ \sum T_{O'} = I_{O'} \ddot{\theta}_r \end{cases} \Rightarrow \begin{cases} -\left(\frac{r}{L}\right) K (\theta_1 - \theta_r) \times \frac{r}{L} = \frac{1}{r} m L^r \ddot{\theta}_1 \\ \left(\frac{r}{L}\right) K (\theta_1 - \theta_r) \frac{r}{L} - \frac{L}{r} K \theta_r \times \frac{L}{r} = \frac{v}{r} m L^r \ddot{\theta}_r \end{cases}$$

$$I_{O'} = \frac{1}{12} m L^r + m \left(\frac{L}{r}\right)^r = \frac{v}{r} m L^r$$

$$\begin{cases} \frac{m}{r} \ddot{\theta}_1 + \frac{9}{16} K \theta_1 - \frac{9}{16} K \theta_r = 0 \\ \frac{vm}{r} \ddot{\theta}_r + \frac{13}{16} K \theta_r - \frac{9}{16} K \theta_1 = 0 \end{cases} ; \begin{cases} \theta_1 = \theta_{1.} \sin(\omega t - \psi) \\ \theta_r = \theta_{r.} \sin(\omega t - \psi) \end{cases}$$

$$\begin{bmatrix} \left(\frac{9}{16} K - \frac{m}{r} \omega^r\right) & -\frac{9}{16} K \\ -\frac{9}{16} K & \left(\frac{13}{16} K - \frac{vm}{r} \omega^r\right) \end{bmatrix} \begin{Bmatrix} \theta_{1.} \\ \theta_{r.} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$\begin{bmatrix} (\Delta 62 / \Delta - 3 / 33 \omega^r) & -\Delta 62 / \Delta \\ -\Delta 62 / \Delta & (112 / \Delta - 1 / 46 \omega^r) \end{bmatrix} \begin{Bmatrix} \theta_{1.} \\ \theta_{r.} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$\frac{\theta_{1.}}{\theta_{r.}} = \frac{\Delta 62 / \Delta}{\Delta 62 / \Delta - 3 / 33 \omega^r}$$

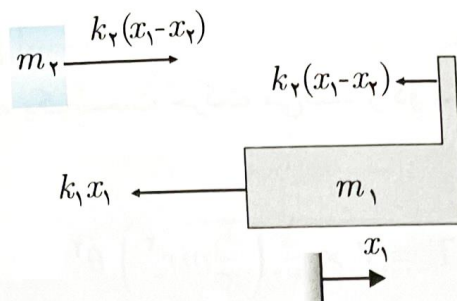
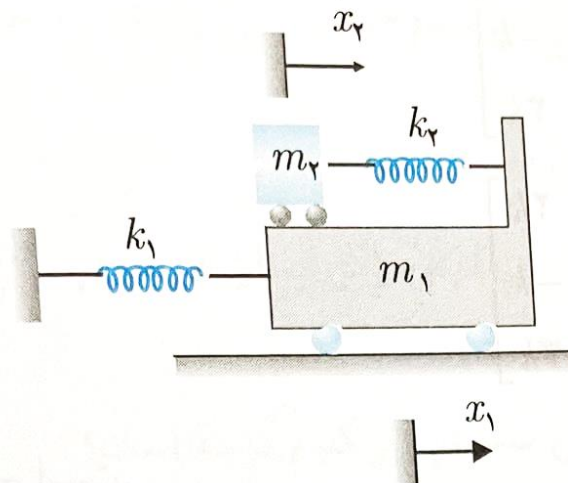
$$(\Delta 62 / \Delta - 3 / 33 \omega^r)(112 / \Delta - 1 / 46 \omega^r) - \Delta 62 / \Delta^r = 0$$

$$4 / 162 \omega^r - 3 \Delta 26 / 11 \omega^r + 143 \cdot 62 / \Delta = 0$$

$$\begin{cases} \omega_{n1}^r = 43 / 12 \\ \omega_{n1}^r = 612 / 27 \end{cases} \rightarrow \begin{cases} \omega_{n1} = 6 / \Delta \gamma \frac{\text{rad}}{\text{s}} \\ \omega_{n2} = 26 / 12 \frac{\text{rad}}{\text{s}} \end{cases}$$

مثال

کدامیک از دستگاه معادلات، حرکت سیستم را به طور کامل مشخص می کند؟

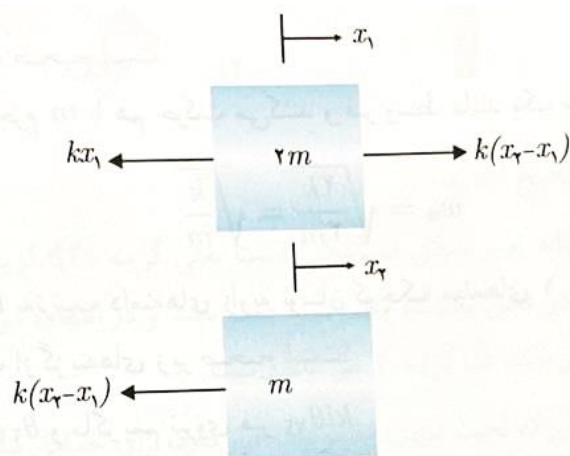
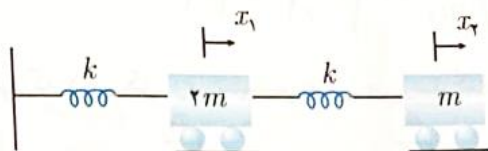


$$\begin{aligned} \sum F = m_1 \ddot{x}_1 &\Rightarrow \begin{cases} -k_1 x_1 - k_2(x_1 - x_2) = m \ddot{x}_1 \\ k_2(x_1 - x_2) = m \ddot{x}_2 \end{cases} \\ \sum F = m_2 \ddot{x}_2 &\Rightarrow \end{aligned}$$

$$\begin{cases} m_1 \ddot{x}_1 + (k_1 + k_2)x_1 - k_2 x_2 = 0 \\ m_2 \ddot{x}_2 + k_2 x_2 - k_2 x_1 = 0 \end{cases}$$

مثال

۹. اگر فرکانس طبیعی اول و دوم سیستم نشان داده شده به ترتیب ω_1 و ω_2 باشند نسبت برابر کدام است؟



$$\sum F = m\ddot{x}_1 \Rightarrow -kx_1 + k(x_2 - x_1) = m\ddot{x}_1$$

$$\sum F = m\ddot{x}_2 \Rightarrow -k(x_2 - x_1) = m\ddot{x}_2$$

$$\begin{cases} 2m\ddot{x}_1 + 2kx_1 - kx_2 = 0 \\ m\ddot{x}_2 + kx_2 - kx_1 = 0 \end{cases}$$

$$k = \begin{bmatrix} 2k & -k \\ -k & k \end{bmatrix} \quad M = \begin{bmatrix} 2m & 0 \\ 0 & m \end{bmatrix}$$

$$|[k] - \omega^2[M]| = 0 \Rightarrow \begin{vmatrix} 2k - 2m\omega^2 & -k \\ -k & k - m\omega^2 \end{vmatrix} = 0$$

$$\Rightarrow 2(k - m\omega^2)^2 - k^2 = 0 \Rightarrow k - m\omega^2 = \pm \frac{k}{\sqrt{2}}$$