mode → complex انتخاب

تغییر حالت shift → mode → complex انتخاب

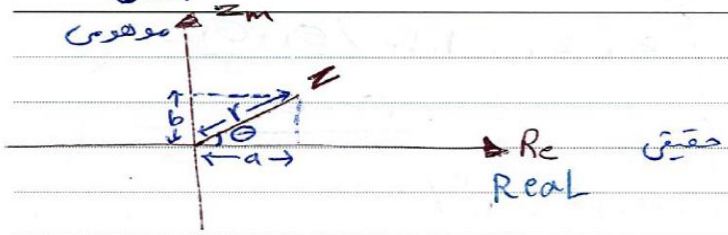
$r \angle \theta$

برای این منظور از علامت ∠ روی دکمه استفاده می کنیم

ca π o 991

Subject:
Year: Month: Date:

اعداد مختلط :



نمایش اعداد مختلط :

$Z = a + jb$ (جبری (دکارتی)

$Z = r\angle\theta$ قطبی

$Z = r(\cos\theta + j\sin\theta)$ مثلثاتی

$Z = re^{j\theta}$ نمایی

تبدیل اعداد دکارتی به قطبی:

$Z = a + jb \Rightarrow r = \sqrt{a^2 + b^2}$

$\theta = \text{Arc tan}\frac{b}{a}$

مثال: عدد $Z = 3 + 4j$ را به صورت قطبی بنویسید.

$r = \sqrt{3^2 + 4^2} = 5$ ، $\theta = \text{Arc tan}\frac{4}{3} = 53^\circ$

$\Rightarrow Z = 5\angle 53^\circ$

تبدیل اعداد قطبی به دکارتی :

$Z = r\angle\theta \Rightarrow a = r\cos\theta$

$b = r\sin\theta$

مثال: عدد $Z = 10\angle 30^\circ$ را به صورت دکارتی بنویسید.

$a = 10\cos 30 = 8,6$

$b = 10\sin 30 = 5$ $\Rightarrow Z = 8,6 + j5$

- چهار عمل اصلی در اعداد مختلط :

$Z_1 = a_1 + jb_1$

$Z_2 = a_2 + jb_2 \Rightarrow$

$Z_1 + Z_2 = (a_1 + a_2) + j(b_1 + b_2)$

$Z_1 - Z_2 = (a_1 - a_2) + j(b_1 - b_2)$

$2Z_1 = 2a_1 + j2b_1$

$$z_1 = r_1 \angle \theta_1 \quad , \quad z_2 = r_2 \angle \theta_2 \Rightarrow \begin{cases} z_1 \cdot z_2 = r_1 r_2 \angle \theta_1 + \theta_2 \\ \frac{z_1}{z_2} = \frac{r_1}{r_2} \angle \theta_1 - \theta_2 \\ (z_1)^n = r_1^n \angle n\theta_1 \end{cases}$$

مزدوج اعداد مختلط :

$$z = r \angle \theta \Rightarrow \bar{z} = r \angle -\theta$$

$$z = a + jb \Rightarrow \bar{z} = a - jb$$

$$z \cdot \bar{z} = r^2 \quad , \quad z\bar{z} = a^2 + b^2$$

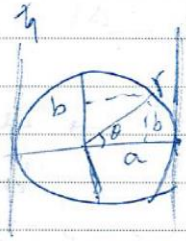
نکته :

$$\sqrt{-1} = j \quad , \quad j^2 = -1 \quad , \quad j^3 = -j \quad , \quad j^4 = 1 \quad , \quad \frac{1}{j} = -j$$

مثال: j^{23} را بدست آورید

$$\begin{array}{r|l} 23 & 4 \\ \hline 20 & 5 \\ \hline (3) & \end{array} \Rightarrow j^{23} = j^3 = -j$$

نکته :



$$z = a + jb \Rightarrow z = r \angle \theta$$

$$z = a \Rightarrow z = a \angle 0^\circ$$

$$z = -a \Rightarrow z = a \angle 180^\circ$$

$$z = aj \Rightarrow z = a \angle 90^\circ$$

$$z = -aj \Rightarrow z = a \angle -90^\circ$$

مثال: اگر $z_1 = 2 + 3j$ ، $z_2 = 5 - 4j$ ، $z_3 = 10 \angle 30^\circ$ ، $z_4 = 2 \angle 60^\circ$ باشد مطلوب است.

الف) $z_1 + z_2 = 7 - j$

ب) $z_1 \cdot z_2 = (2+3j)(5-4j) = 10 - 8j + 15j + 12$

ج) $\frac{z_1}{z_2} = \frac{2+3j}{5-4j} \times \frac{5+4j}{5+4j} = \frac{10+8j+15j-12}{25+16} = \frac{-2}{41} + j\frac{23}{41}$

د) $= \frac{z_4}{z_3} = \frac{2 \angle 60^\circ}{10 \angle 30^\circ} = 0.2 \angle 30^\circ$

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هـ) $Z_4^5 = (2\angle 60)^5 = 32\angle 300^\circ$

و) $\bar{Z}_2 = 5+4j$

نکته: $\sqrt[n]{r\angle\theta} = \sqrt[n]{r}\angle\frac{2k\pi+\theta}{n}$ ، $k=\{0,1,2,\dots,n-1\}$

مثال: معادلات زیر را حل کنید.

الف) $x^5-32=0$

$x^5=32 \Rightarrow \sqrt[5]{x^5}=\sqrt[5]{32\angle 0} \Rightarrow x=\sqrt[5]{32}\angle\frac{2k\pi+0}{5}$

$k=0 \Rightarrow x_1=2\angle 0$
$k=1 \Rightarrow x_2=2\angle 72^\circ$
$k=2 \Rightarrow x_3=2\angle 144^\circ$
$k=3 \Rightarrow x_4=2\angle 216^\circ$
$k=4 \Rightarrow x_5=2\angle 288^\circ$

ب) $x^2+4=0 \Rightarrow x^2=-4 \Rightarrow \sqrt{x^2}=\sqrt{4\angle 180} \Rightarrow x=2\angle\frac{2k\pi+180}{2}$

$k=0 \Rightarrow x_1=2\angle 90$
$k=1 \Rightarrow x_2=2\angle 270^\circ$

R

$jxL=j\omega L$

$-jxc=\frac{1}{j\omega c}$

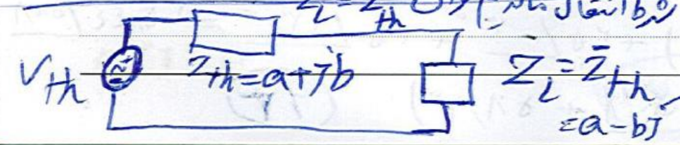
R x_c

$Z=R-jX_c=R+\frac{1}{j\omega c}$

R x_L

$R+jX_L=R+j\omega L$

شرط انتقال ماکزیمم توان $Z_L=\bar{Z}_{th}$



اگر سلف و خازن از بین می‌برد (در شکل خازنی)

$r\angle\theta \Rightarrow r\cos(\omega t+\theta) \xrightarrow{\theta=0} r\cos\omega t$

$r\angle -90+\theta \Rightarrow r\cos(\omega t+\theta-90^\circ) = r\sin(\omega t+\theta)$ $\xrightarrow{\theta=0}$ سینوسی

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تبدیل زمان به فیزور:

زمان ↓ فیزور ↓

$r\cos(\omega t) = r\angle 0$

$r\cos(\omega t+\theta) = r\angle\theta$

$r\sin\omega t = r\angle -90^\circ$

$r\sin(\omega t+\theta) = r\angle -90^\circ+\theta$

توان:

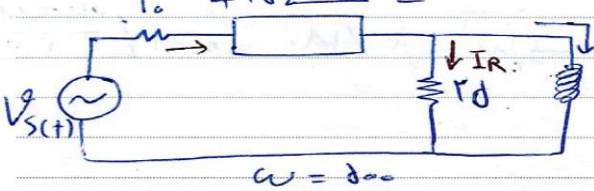
$S = P + jQ$ → حواله (V.A.R)

ظاهری (V.A) حقیقی (W)

$S = V_e \cdot I_e^*$ مزدوج

$S = \frac{1}{2} V_m \cdot I_m^*$

ص ۲۷ سوال ۳۲ (3)



$2.5\angle 40^\circ$ A

$V_L = j\omega L I$

$V_L = (2.5\angle 40^\circ)(j(500)(2\times10^{-2})) = 25\angle 130^\circ$

$V_{S(t)} = ?$

$I_R = \frac{25\angle 130^\circ}{25} = 1\angle 130^\circ$

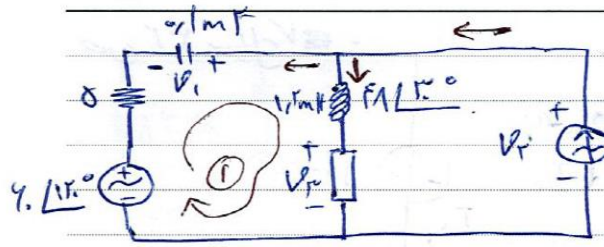
$I_t = 1\angle 130^\circ + 2.5\angle 40^\circ = 2.49\angle 41.8^\circ$

$V_S = (1)(2.49\angle 41.8^\circ) + 25\angle -30^\circ + 25\angle 130^\circ = 35.64\angle 51.9^\circ$

$V_{S(t)} = 35.64\cos(500t + 51.9^\circ)$ (۴۴)

HELIA

ص ۲۷ سوال ۳۳ :



$\omega = 500$

$v_1(t) = ?$
$v_2(t) = ?$
$v_3(t) = ?$

$$I_L = \frac{48\angle 30^\circ}{j(500)(12\times 10^{-3})} = 8\angle -60^\circ$$

$$I_C = 10\angle 0^\circ - 8\angle -60^\circ = 9.12\angle 49^\circ$$

$$V_1 = (9.12\angle 49^\circ)\left(\frac{1}{j(500)(0.1\times 10^{-3})}\right) = 18.22\angle -41^\circ$$

$$v_1(t) = 18.22\cos(500t - 41^\circ)$$

$$① \rightarrow -60\angle 120^\circ - 5(9.12\angle 49^\circ) - 18.22\angle -41^\circ + 48\angle 30^\circ + V_3 = 0$$

$$V_3 = 60\angle 120^\circ + 45.6\angle 49^\circ + 18.22\angle -41^\circ - 48\angle 30^\circ$$

$$V_3 = 57.4\angle 111.7^\circ$$

$$v_3(t) = 57.4\cos(500t + 111.7^\circ)$$

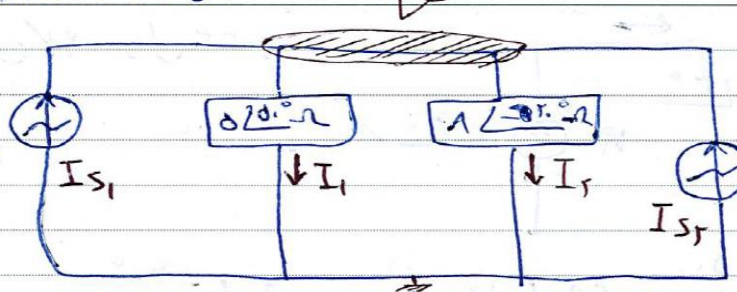
$$V_2 = 48\angle 30^\circ + 57.4\angle 111.7^\circ = 75.8\angle 79.4^\circ$$

$$v_2(t) = 75.8\cos(500t + 79.4^\circ)$$

Year: Month: Date:

ص ۳۲ سوال ۷ :

ص ۳۲ : ۱۲/۱۴/۸



$$-10 + \frac{V}{5\angle 50^\circ} + \frac{V}{8\angle -30^\circ} - j10 = 0 \Rightarrow 0.2V\angle -50^\circ + 0.125V\angle 30^\circ = 10 + j10$$

$$V = \frac{10 + j10}{0.2\angle -50^\circ + 0.125\angle 30^\circ} = 54.33\angle 49.17^\circ$$

$$I_1 = \frac{54.33\angle 49.17^\circ}{5\angle 50^\circ} = 10.87\angle -19.17^\circ$$

$$I_2 = \frac{54.33\angle 49.17^\circ}{8\angle -30^\circ} = 6.79\angle 79.17^\circ$$

P_{av}

منبع:
$$S_1 = \frac{1}{2}(54.33\angle 49.17^\circ)(10\angle 50^\circ) = 95.33 + j36.2$$
$$S_2 = \frac{1}{2}(54.33\angle 49.17^\circ)(10\angle -90^\circ) = -95.33 - j36.2$$

مصرف:
$$S_1 = \frac{1}{2}(54.33\angle 49.17^\circ)(10.87\angle -19.17^\circ) = 147.41 + j141.78$$
$$S_2 = \frac{1}{2}(54.33\angle 49.17^\circ)(6.79\angle 79.17^\circ) = 184.24 - j141.78$$

P_{av} منبع $= 95.33 + 36.2 = 131.53$

P_{av} مصرف کننده $= 147.41 + 184.24 = 331.65$

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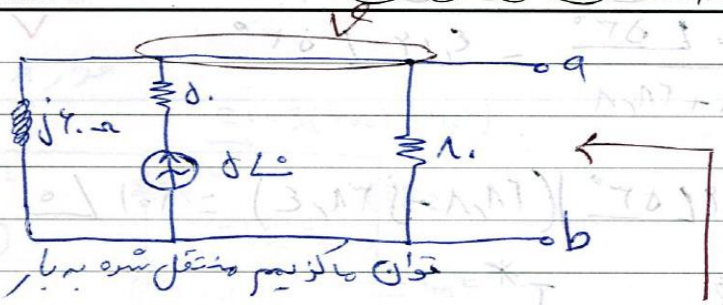
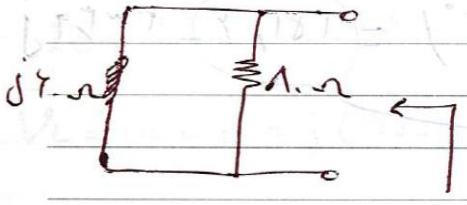
حل ۳۲ سوال ۱۵:



توان ماکزیمم منتقل شده به بار

$Z_{eq} = ?$

بدست آوردن Z_{eq}: $Z_{eq} = Z_{th} = Z_N$



$$Z_{th} = \frac{(10)(j20)}{10 + j20} = 8 + j4 \quad \Omega$$

بدست آوردن V_{th}:

$$\frac{V_\varphi}{j20} - 5\angle 0^\circ + \frac{V_\varphi}{10} = 0$$ طرفین ضرب ۲۰

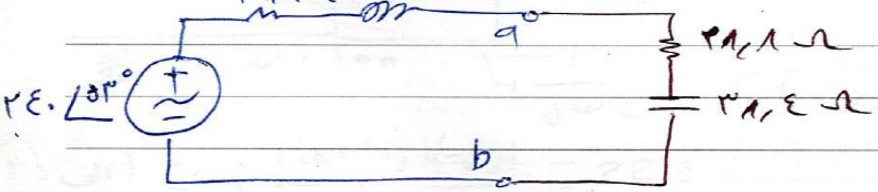
$$-4V_\varphi j + 2V_\varphi = 100\angle 0 \Rightarrow V_{th} = \frac{100}{2 - 4j} = 20\angle 63^\circ$$

شرط حداکثر توان انتقال به بار به صورت زیر است:

$$Z_L = \overline{Z}_{th}$$

$$Z_L = \overline{Z}_N$$

مدار معادل تونن:



$$Z_L = 8 - j4 \quad \Omega$$

Raz

$$I_{L_{max}} = \frac{240 \angle 53^\circ}{28.8 + 28.8} = 4.17 \angle 53^\circ$$

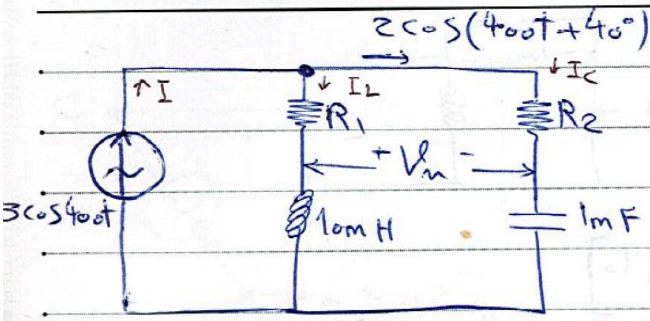
$$V_{L_{max}} = (4.17 \angle 53^\circ)(28.8 - j28.8) = 201 \angle 0$$

$$S = \frac{1}{2} V_{L_{max}}^{*} \cdot I_{L_{max}}^{*}$$

$$S = \frac{1}{2}(201 \angle 0)(4.17 \angle -53^\circ) = 251.2 - j334.9$$

$$P_{L_{max}} = 251.2 \text{ W}$$

: 35 سؤال 27 ص



$I_L = I - I_C = 3\angle 0 - 2\angle 40° = 1.95\angle -41.2°$

$V_L = j\omega L I = j(400)(10\times10^{-3})(1.95\angle -41.2°) = 7.8\angle 48.8°$

$V_C = \frac{1}{j\omega C} I = \frac{1}{j(400)(1\times10^{-3})} \times 2\angle 40° = 5\angle -50°$

$V_n = V_L - V_C = 7.8\angle 48.8 - 5\angle -50° = 9.8\angle 78.7°$

$V_n(t) = 9.8\cos(400t + 78.7°)$ ✓

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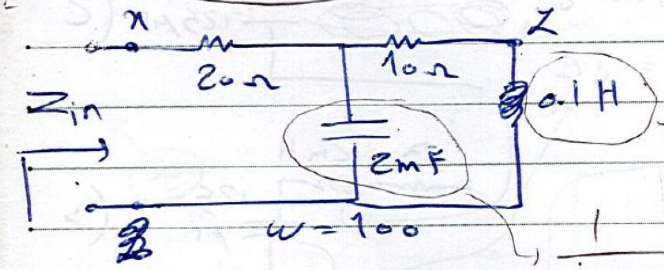


$\rightarrow j\omega L = 10j\ \Omega$

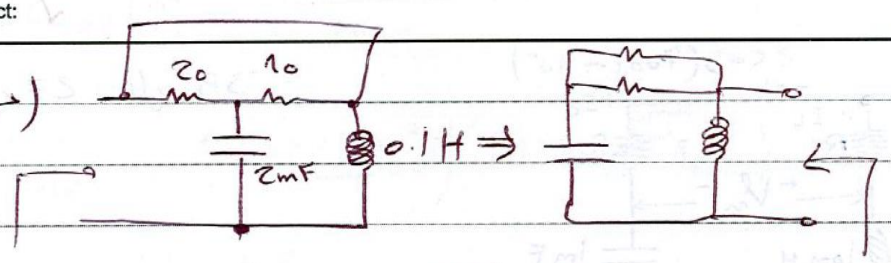
$\frac{1}{j\omega C} = \frac{1}{j(100)(2\times10^{-3})} = -5j\ \Omega$

الحل) $20 + \frac{(10+10j)(-j5)}{(10+10j)-5j} = 22.8\angle -15.2°$

Subject:

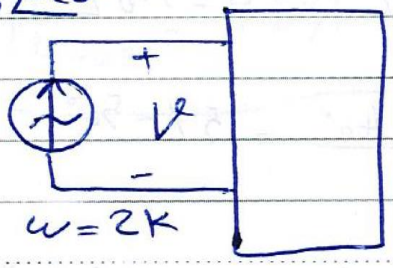
(2)

ب)

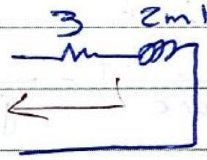


$$Z_{in} = \frac{[20\|10 + (-j5)][j10]}{20\|10 + (-j5) + j10} = 10\angle 16.2^\circ$$ ✓

① ص 27 سؤال 40 :

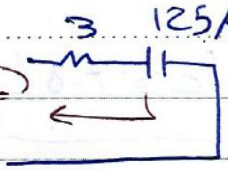


(الف



$j\omega L = j(2000)(2\times10^{-3}) = j4$

(ب



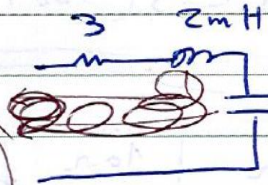
$\frac{1}{j\omega C} = \frac{1}{j(2000)(125\times10^{-6})}$

V =

الف) $(3\angle -20)(3+j4)$

$= 15\angle 33^\circ$

(ج



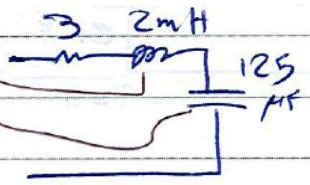
$j\omega L = j8$

$\frac{1}{j\omega C} = -j2$

ب) $V = (3\angle -20)(3-j4)$

$= 15\angle -73^\circ$

(د



ج) $V = (3\angle -20)(3+j4-j4) = 9\angle -20^\circ$

د) $V = (3\angle -20)(3+j8-j2) = 20.12\angle 43^\circ$

Kimia Stationery Collection

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